CHAPTER 21

Probability

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. For a biased die the probabilities for the different faces to turn up are given below :

Face	1	2	3	4	5	6
Prob.	0.1	0.32	0.21	0.15	0.05	0.17

This die is tossed and you are told that either face 1 or face 2 has turned up. Then the probability that it is face 1 is .... *(1981 - 2 Marks)*

2. $P(A \cup B) = P(A \cap B)$ if and only if the relation between $P(A)$ and $P(B)$ is .... *(1985 - 2 Marks)*

3. A box contains 100 tickets numbered 1, 2, ..., 100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. The minimum number on them is 5 with probability .... *(1985 - 2 Marks)*

4. If $\frac{1+3p}{3}$, $\frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probabilities of three mutually exclusive events, then the set of all values of p is .... *(1986 - 2 Marks)*

5. Urn A contains 6 red and 4 black balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is drawn at random from urn B and placed in urn A. If one ball is now drawn at random from urn A, the probability that it is found to be red is .... *(1988 - 2 Marks)*

6. A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. Then the probability that 5 comes before 7 is .... *(1989 - 2 Marks)*

7. Let A and B be two events such that $P(A) = 0.3$ and $P(A \cup B) = 0.8$. If A and B are independent events then $P(B) =$ .... *(1990 - 2 Marks)*

8. If the mean and the variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than one is equal to .... *(1991 - 2 Marks)*

9. Three faces of a fair die are yellow, two faces red and one blue. The die is tossed three times. The probability that the colours, yellow, red and blue, appear in the first, second and the third tosses respectively is ....*(1992 - 2 Marks)*

10. If two events A and B are such that $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^c) = 0.5$, then $P(B/(A \cup B^c)) =$ .... *(1994 - 2 Marks)*

B True / False

1. If the letters of the word "Assassin" are written down at random in a row, the probability that no two S's occur together is 1/35 *(1983 - 1 Mark)*

2. If the probability for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails is 0.5. *(1989 - 1 Mark)*

C MCQs with One Correct Answer

1. Two fair dice are tossed. Let x be the event that the first die shows an even number and y be the event that the second die shows an odd number. The two events x and y are : *(1979)*
   (a) Mutually exclusive
   (b) Independent and mutually exclusive
   (c) Dependent
   (d) None of these.

2. Two events A and B have probabilities 0.25 and 0.50 respectively. The probability that both A and B occur simultaneously is 0.14. Then the probability that neither A nor B occurs is *(1980)*
   (a) 0.39 (b) 0.25 (c) 0.11 (d) none of these

3. The probability that an event A happens in one trial of an experiment is 0.4. Three independent trials of the experiment are performed. The probability that the event A happens at least once is *(1980)*
   (a) 0.936 (b) 0.784 (c) 0.904 (d) none of these

4. If A and B are two events such that $P(A) > 0$, and $P(B) \neq 1$, then $P\left(\frac{\overline{A}}{\overline{B}}\right)$ is equal to *(1982 - 2 Marks)*
   (a) $1 - P(\frac{A}{B})$ (b) $1 - P(\frac{\overline{A}}{B})$
   (c) $\frac{1 - P(A \cup B)}{P(\overline{B})}$ (d) $\frac{P(\overline{A})}{P(\overline{B})}$
   (Here $\overline{A}$ and $\overline{B}$ are complements of A and B respectively).

5. Fifteen coupons are numbered 1, 2 ..... 15, respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9, is *(1983 - 1 Mark)*
   (a) $\left(\frac{9}{16}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$ (c) $\left(\frac{3}{5}\right)^7$ (d) none of these

6. Three identical dice are rolled. The probability that the same number will appear on each of them is *(1984 - 2 Marks)*
(a) 1/6 (b) 1/36 (c) 1/18 (d) 3/28

7. A box contains 24 identical balls of which 12 are white and 12 are black. The balls are drawn at random from the box one at a time with replacement. The probability that a white ball is drawn for the 4th time on the 7th draw is *(1984 - 2 Marks)*
(a) 5/64 (b) 27/32 (c) 5/32 (d) 1/2

8. One hundred identical coins, each with probability, *p*, of showing up heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of *p* is *(1988 - 2 Marks)*
(a) 1/2 (b) 49/101 (c) 50/101 (d) 51/101.

9. India plays two matches each with West Indies and Australia. In any match the probabilities of India getting, points 0, 1 and 2 are 0.45, 0.05 and 0.50 respectively. Assuming that the outcomes are independent, the probability of India getting at least 7 points is *(1992 - 2 Marks)*
(a) 0.8750 (b) 0.0875 (c) 0.0625 (d) 0.0250

10. An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Out of four face values obtained, the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5, is then: *(1993 - 1 Mark)*
(a) 16/81 (b) 1/81 (c) 80/81 (d) 65/81

11. Let *A*, *B*, *C* be three mutually independent events. Consider the two statements S_1 and S_2
S_1 : *A* and $B \cup C$ are independent
S_2 : *A* and $B \cap C$ are independent
Then, *(1994)*
(a) Both S_1 and S_2 are true
(b) Only S_1 is true
(c) Only S_2 is true
(d) Neither S_1 nor S_2 is true

12. The probability of India winning a test match against west Indies is 1/2. Assuming independence from match to match the probability that in a 5 match series India's second win occurs at third test is *(1995S)*
(a) 1/8 (b) 1/4 (c) 1/2 (d) 2/3

13. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with three vertices is equilateral, equals *(1995S)*
(a) 1/2 (b) 1/5 (c) 1/10 (d) 1/20

14. For the three events *A*, *B*, and *C*, *P* (exactly one of the events *A* or *B* occurs) = *P* (exactly one of the two events *B* or *C* occurs) = *P*(exactly one of the events *C* or *A* occurs) = *p* and *P* (all the three events occur simultaneously) = p^2, where $0 < p < 1/2$. Then the probability of at least one of the three events *A*, *B* and *C* occurring is *(1996 - 2 Marks)*
(a) $\frac{3p+2p^2}{2}$ (b) $\frac{p+3p^2}{4}$
(c) $\frac{p+3p^2}{2}$ (d) $\frac{3p+2p^2}{4}$

15. If the integers *m* and *n* are chosen at random from 1 to 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 equals *(1999 - 2 Marks)*
(a) 1/4 (b) 1/7 (c) 1/8 (d) 1/49

16. Two numbers are selected randomly from the set *S* = {1, 2, 3, 4, 5, 6} without replacement one by one. The probability that minimum of the two numbers is less than 4 is *(2003S)*
(a) 1/15 (b) 14/15 (c) 1/5 (d) 4/5

17. If $P(B) = \frac{3}{4}, P(A \cap B \cap \bar{C}) = \frac{1}{3}$ and *(2003S)*

$P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{3}$, then $P(B \cap C)$ is
(a) 1/12 (b) 1/6 (c) 1/15 (d) 1/9

18. If three distinct numbers are chosen randomly from the first 100 natural numbers, then the probability that all three of them are divisible by both 2 and 3 is *(2004S)*
(a) 4/25 (b) 4/35 (c) 4/33 (d) 4/1155

19. A six faced fair dice is thrown until 1 comes, then the probability that 1 comes in even no. of trials is *(2005S)*
(a) 5/11 (b) 5/6 (c) 6/11 (d) 1/6

20. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is *(2007 -3 marks)*
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$

21. Let E^c denote the complement of an event *E*. Let *E*, *F*, *G* be pairwise independent events with $P(G) > 0$ and $P(E \cap F \cap G) = 0$. Then $P(E^c \cap F^c | G)$ equals *(2007 -3 marks)*
(a) $P(E^c) + P(F^c)$ (b) $P(E^c) - P(F^c)$
(c) $P(E^c) - P(F)$ (d) $P(E) - P(F^c)$

22. An experiment has 10 equally likely outcomes. Let *A* and *B* be non-empty events of the experiment. If *A* consists of 4 outcomes, the number of outcomes that *B* must have so that *A* and *B* are independent, is *(2008)*
(a) 2, 4 or 8 (b) 3, 6 or 9 (c) 4 or 8 (d) 5 or 10

23. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is *(2010)*
(a) $\frac{1}{18}$ (b) $\frac{1}{9}$ (c) $\frac{2}{9}$ (d) $\frac{1}{36}$

24. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is *(2010)*
(a) $\frac{3}{5}$ (b) $\frac{6}{7}$ (c) $\frac{20}{23}$ (d) $\frac{9}{20}$

25. Four fair dice D_1, D_2, D_3 and D_4; each having six faces numbered 1, 2, 3, 4, 5 and 6 are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is *(2012)*

(a) $\frac{91}{216}$ (b) $\frac{108}{216}$ (c) $\frac{125}{216}$ (d) $\frac{127}{216}$

26. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is *(JEE Adv. 2014)*

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

27. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that P (computer turns out to be defective given that it is produced in plant T_1)
= 10P (computer turns out to be defective given that it is produced in plant T_2),
where P(E) denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is *(JEE Adv. 2016)*

(a) $\frac{36}{73}$ (b) $\frac{47}{79}$ (c) $\frac{78}{93}$ (d) $\frac{75}{83}$

D MCQs with One or More than One Correct

1. If M and N are any two events, the probability that exactly one of them occurs is *(1984 - 3 Marks)*

(a) $P(M)+P(N)-2P(M\cap N)$

(b) $P(M)+P(N)-P(M\cap N)$

(c) $P(M^c)+P(N^c)-2P(M^c\cap N^c)$

(d) $P(M\cap N^c)+P(M^c\cap N)$

2. A student appears for tests I, II and III. The student is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in tests I, II and III are p, q and $\frac{1}{2}$ respectively. If the probability that the student is successful is $\frac{1}{2}$, then *(1986 - 2 Marks)*

(a) $p=q=1$ (b) $p=q=\frac{1}{2}$

(c) $p=1, q=0$ (d) $p=1, q=\frac{1}{2}$

(e) none of these

3. The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\bar{A})+P(\bar{B})$ is *(1987 - 2 Marks)*

(a) 0.4 (b) 0.8 (c) 1.2 (d) 1.4

(e) none

(Here $\bar{A}$ and $\bar{B}$ are complements of A and B, respectively).

4. For two given events A and B, $P(A\cap B)$ *(1988 - 2 Marks)*

(a) not less than $P(A)+P(B)-1$

(b) not greater than $P(A)+P(B)$

(c) equal to $P(A)+P(B)-P(A\cup B)$

(d) equal to $P(A)+P(B)+P(A\cup B)$

5. If E and F are independent events such that $0<P(E)<1$ and $0<P(F)<1$, then *(1989 - 2 Marks)*

(a) E and F are mutually exclusive

(b) E and F^c (the complement of the event F) are independent

(c) E^c and F^c are independent

(d) $P(E|F)+P(E^c|F)=1$.

6. For any two events A and B in a sample space *(1991 - 2 Marks)*

(a) $P(A/B)\geq\frac{P(A)+P(B)-1}{P(B)}, P(B)\neq 0$ is always true

(b) $P(A\cap\bar{B}) = P(A)-P(A\cap B)$ does not hold

(c) $P(A\cup B) = 1-P(\bar{A})P(\bar{B})$, if A and B are independent

(d) $P(A\cup B) = 1-P(\bar{A})P(\bar{B})$, if A and B are disjoint.

7. E and F are two independent events. The probability that both E and F happen is 1/12 and the probability that neither E nor F happens is 1/2. Then, *(1993 - 2 Marks)*

(a) $P(E)=1/3, P(F)=1/4$

(b) $P(E)=1/2, P(F)=1/6$

(c) $P(E)=1/6, P(F)=1/2$

(d) $P(E)=1/4, P(F)=1/3$

8. Let $0<P(A)<1, 0<P(B)<1$ and $P(A\cup B) = P(A)+P(B)-P(A)P(B)$ then *(1995S)*

(a) $P(B/A)=P(B)-P(A)$

(b) $P(A'-B')=P(A')-P(B')$

(c) $P(A\cup B)' = P(A')P(B')$

(d) $P(A/B)=P(A)$

9. If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is *(1998 - 2 Marks)*

(a) 13/32 (b) 1/4

(c) 1/32 (d) 3/16

10. If $\overline{E}$ and $\overline{F}$ are the complementary events of events E and F respectively and if $0 < P(F) < 1$, then *(1998 - 2 Marks)*

(a) $P(E/F) + P(\overline{E}/F) = 1$

(b) $P(E/F) + P(E/\overline{F}) = 1$

(c) $P(\overline{E}/F) + P(E/\overline{F}) = 1$

(d) $P(E/\overline{F}) + P(\overline{E}/\overline{F}) = 1$

11. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then the probability that only two tests are needed is *(1998 - 2 Marks)*

(a) 1/3 (b) 1/6 (c) 1/2 (d) 1/4

12. If E and F are events with $P(E) \le P(F)$ and $P(E \cap F) > 0$, then *(1998 - 2 Marks)*

(a) occurrence of $E \Rightarrow$ occurrence of F

(b) occurrence of $F \Rightarrow$ occurrence of E

(c) non-occurrence of $E \Rightarrow$ non-occurrence of F

(d) none of the above implications holds

13. A fair coin is tossed repeatedly. If the tail appears on first four tosses, then the probability of the head appearing on the fifth toss equals *(1998 - 2 Marks)*

(a) 1/2 (b) 1/32 (c) 31/32 (d) 1/5

14. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals *(1998 - 2 Marks)*

(a) 1/2 (b) 7/15 (c) 2/15 (d) 1/3

15. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c, respectively. Of these subjects, the student has a 75% chance of passing in at least one, a 50% chance of passing in at least two, and a 40% chance of passing in exactly two. Which of the following relations are true? *(1999 - 3 Marks)*

(a) $p + m + c = 19/20$ (b) $p + m + c = 27/20$

(c) $pmc = 1/10$ (d) $pmc = 1/4$

16. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If $P(T)$ denotes the probability of occurrence of the event T, then *(2011)*

(a) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$ (b) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$

(c) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$ (d) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$

17. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is(are) true ? *(2012)*

(a) $P\left[X_1^c | X\right] = \frac{3}{16}$

(b) P [Exactly two engines of the ship are functioning $|X] = \frac{7}{8}$

(c) $P\left[X | X_2\right] = \frac{5}{16}$ (d) $P\left[X | X_1\right] = \frac{7}{16}$

18. Let X and Y be two events such that $P(X|Y) = \frac{1}{2}$, $P(Y/X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$. Which of the following is (are) correct ? *(2012)*

(a) $P(X \cup Y) = \frac{2}{3}$

(b) X and Y are independent

(c) X and Y are not independent

(d) $P\left(X^c \cap Y\right) = \frac{1}{3}$

19. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is *(JEE Adv. 2013)*

(a) $\frac{235}{256}$ (b) $\frac{21}{256}$ (c) $\frac{3}{256}$ (d) $\frac{253}{256}$

E Subjective Problems

1. Balls are drawn one-by-one without replacement from a box containing 2 black, 4 white and 3 red balls till all the balls are drawn. Find the probability that the balls drawn are in the order 2 black, 4 white and 3 red. *(1978)*

2. Six boys and six girls sit in a row randomly. Find the probability that

(i) the six girls sit together

(ii) the boys and girls sit alternately. *(1979)*

3. An anti -aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that the gun hits the plane ? *(1981 - 2 Marks)*

4. A and B are two candidates seeking admission in IIT. The probability that A is selected is 0.5 and the probability that both A and B are selected is atmost 0.3. Is it possible that the probability of B getting selected is 0.9 ? *(1982 - 2 Marks)*

5. Cards are drawn one by one at random from a well-shuffled full pack of 52 playing cards until 2 aces are obtained for the first time. If N is the number of cards required to be drawn, then show that $P_r\{N=n\}=\dfrac{(n-1)(52-n)(51-n)}{50\times 49\times 17\times 13}$ where $2\le n\le 50$ *(1983 - 3 Marks)*

6. A, B, C are events such that *(1983 - 2 Marks)*
$P(A)=0.3, P(B)=0.4, P(C)=0.8$
$P(AB)=0.08, P(AC)=0.28;\ \ P(ABC)=0.09$
If $P(A\cup B\cup C)\ge 0.75$, then show that $P(BC)$ lies in the interval $0.23\le x\le 0.48$

7. In a certain city only two newspapers A and B are published, it is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. It is also known that 30% of those who read A but not B look into advertisements and 40% of those who read B but not A look into advertisements while 50% of those who read both A and B look into advertisements. What is the percentage of the population that reads an advertisement? *(1984 - 4 Marks)*

8. In a multiple-choice question there are four alternative answers, of which one or more are correct. A candidate will get marks in the question only if he ticks the correct answers. The candidate decides to tick the answers at random, if he is allowed upto three chances to answer the questions, find the probability that he will get marks in the questions. *(1985 - 5 Marks)*

9. A lot contains 20 articles. The probability that the lot contains exactly 2 defective articles is 0.4 and the probability that the lot contains exactly 3 defective articles is 0.6. Articles are drawn from the lot at random one by one without replacement and are tested till all defective articles are found. What is the probability that the testing procedure ends at the twelth testing. *(1986 - 5 Marks)*

10. A man takes a step forward with probability 0.4 and backwards with probability 0.6 Find the probability that at the end of eleven steps he is one step away from the starting point. *(1987 - 3 Marks)*

11. A box contains 2 fifty paise coins, 5 twenty five paise coins and a certain fixed number N (≥ 2) of ten and five paise coins. Five coins are taken out of the box at random. Find the probability that the total value of these 5 coins is less than one rupee and fifty paise. *(1988 - 3 Marks)*

12. Suppose the probability for A to win a game against B is 0.4. If A has an option of playing either a "best of 3 games" or a "best of 5 games" match against B, which option should be choose so that the probability of his winning the match is higher? (No game ends in a draw). *(1989 - 5 Marks)*

13. A is a set containing n elements. A subset P of A is chosen at random. The set A is reconstructed by replacing the elements of P. A subset Q of A is again chosen at random. Find the probability that P and Q have no common elements. *(1990 - 5 Marks)*

14. In a test an examine either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he make a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct given that he copied it, is 1/8. Find the probability that he knew the answer to the question given that he correctly answered it. *(1991 - 4 Marks)*

15. A lot contains 50 defective and 50 non defective bulbs. Two bulbs are drawn at random, one at a time, with replacement. The events A, B, C are defined as *(1992 - 6 Marks)*
A = (the first bulb is defective)
B = (the second bulb is non-defective)
C = (the two bulbs are both defective or both non defective)
Determine whether
(i) A, B, C are pairwise independent
(ii) A, B, C are independent

16. Numbers are selected at random, one at a time, from the two-digit numbers 00, 01, 02....., 99 with replacement. An event E occurs if only if the product of the two digits of a selected number is 18. If four numbers are selected, find probability that the event E occurs at least 3 times. *(1993 - 5 Marks)*

17. An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2, 3, 4,.....12 is picked and the number on the card is noted. What is the probability that the noted number is either 7 or 8? *(1994 - 5 Marks)*

18. In how many ways three girls and nine boys can be seated in two vans, each having numbered seats, 3 in the front and 4 at the back? How many seating arrangements are possible if 3 girls should sit together in a back row on adjacent seats? Now, if all the seating arrangements are equally likely, what is the probability of 3 girls sitting together in a back row on adjacent seats? *(1996 - 5 Marks)*

19. If p and q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, with replacement, determine the probability that the roots of the equation $x^2+px+q=0$ are real. *(1997 - 5 Marks)*

20. Three players, A, B and C, toss a coin cyclically in that order (that is A, B, C, A, B, C, A, B, ...) till a head shows. Let p be the probability that the coin shows a head. Let α, β and γ be, respectively, the probabilities that A, B and C gets the first head. Prove that $\beta=(1-p)\alpha$. Determine α, β and γ (in terms of p). *(1998 - 8 Marks)*

21. Eight players $P_1, P_2, \ldots\ldots P_8$ play a knock-out tournament. It is known that whenever the players P_i and P_j play, the player P_i will win if $i<j$. Assuming that the players are paired at random in each round, what is the probability that the player P_4 reaches the final? *(1999 - 10 Marks)*

22. A coin has probability p of showing head when tossed. It is tossed n times. Let p_n denote the probability that no two (or more) consecutive heads occur. Prove that $p_1=1$, $p_2=1-p^2$ and $p_n=(1-p).\ p_{n-1}+p(1-p)\,p_{n-2}$ for all $n\ge 3$. *(2000 - 5 Marks)*

23. An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white? *(2001 - 5 Marks)*

24. An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6, is thrown n times and the list of n numbers showing up is noted. What is the probability that, among the numbers 1, 2, 3, 4, 5, 6, only three numbers appear in this list? *(2001 - 5 Marks)*

25. A box contains N coins, m of which are fair and the rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. What is the probability that the coin drawn is fair? *(2002 - 5 Marks)*

26. For a student to qualify, he must pass at least two out of three exams. The probability that he will pass the 1st exam is p. If he fails in one of the exams then the probability of his passing in the next exam is $\frac{p}{2}$ otherwise it remains the same. Find the probability that he will qualify. *(2003 - 2 Marks)*

27. A is targeting to B, B and C are targeting to A. Probability of hitting the target by A, B and C are $\frac{2}{3}, \frac{1}{2}$ and $\frac{1}{3}$ respectively.

If A is hit then find the probability that B hits the target and C does not. *(2003 - 2 Marks)*

28. A and B are two independent events. C is event in which exactly one of A or B occurs. Prove that

$P(C) \geq P(A \cup B)P(\bar{A} \cap \bar{B})$ *(2004 - 2 Marks)*

29. A box contains 12 red and 6 white balls. Balls are drawn from the box one at a time without replacement. If in 6 draws there are at least 4 white balls, find the probability that exactly one white is drawn in the next two draws. (binomial coefficients can be left as such) *(2004 - 4 Marks)*

30. A person goes to office either by car, scooter, bus or train, the probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reached office in time, then what is the probability that he travelled by a car. *(2005 - 2 Marks)*

G Comprehension Based Questions

PASSAGE - 1

There are n urns, each of these contain $n+1$ balls. The ith urn contains i white balls and $(n+1-i)$ red balls. Let u_i be the event of selecting ith urn, $i = 1, 2, 3$ ...., n and w the event of getting a white ball.

1. If $P(u_i) \propto i$, where $i = 1, 2, 3, ..., n$, then $\lim_{n \to \infty} P(w) =$ *(2006 - 5M, –2)*

(a) 1 (b) 2/3 (c) 3/4 (d) 1/4

2. If $P(u_i) = c$, (a constant) then $P(u_n/w) =$ *(2006 - 5M, –2)*

(a) $\frac{2}{n+1}$ (b) $\frac{1}{n+1}$ (c) $\frac{n}{n+1}$ (d) $\frac{1}{2}$

3. Let $P(u_i) = \frac{1}{n}$, if n is even and E denotes the event of choosing even numbered urn, then the value of $P(w/E)$ is *(2006 - 5M, –2)*

(a) $\frac{n+2}{2n+1}$ (b) $\frac{n+2}{2(n+1)}$ (c) $\frac{n}{n+1}$ (d) $\frac{1}{n+1}$

PASSAGE - 2

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required. *(2009)*

4. The probability that $X = 3$ equals

(a) $\frac{25}{216}$ (b) $\frac{25}{36}$ (c) $\frac{5}{36}$ (d) $\frac{125}{216}$

5. The probability that $X \geq 3$ equals

(a) $\frac{125}{216}$ (b) $\frac{25}{36}$ (c) $\frac{5}{36}$ (d) $\frac{25}{216}$

6. The conditional probability that $X \geq 6$ given $X > 3$ equals

(a) $\frac{125}{216}$ (b) $\frac{25}{216}$ (c) $\frac{5}{36}$ (d) $\frac{25}{36}$

PASSAGE - 3

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2. *(2011)*

7. The probability of the drawn ball from U_2 being white is

(a) $\frac{13}{30}$ (b) $\frac{23}{30}$ (c) $\frac{19}{30}$ (d) $\frac{11}{30}$

8. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(a) $\frac{17}{23}$ (b) $\frac{11}{23}$ (c) $\frac{15}{23}$ (d) $\frac{12}{23}$

PASSAGE - 4

A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 5 black balls.

9. If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, the probability that all 3 drawn balls are of the same colour is *(JEE Adv. 2013)*

(a) $\frac{82}{648}$ (b) $\frac{90}{648}$ (c) $\frac{558}{648}$ (d) $\frac{566}{648}$

10. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is

(a) $\frac{116}{181}$ (b) $\frac{126}{181}$ (c) $\frac{65}{181}$ (d) $\frac{55}{181}$

PASSAGE - 5

Box 1 contains three cards bearing numbers 1, 2, 3; box 2 contains five cards bearing numbers 1, 2, 3, 4, 5; and box 3 contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let x_i be number on the card drawn from the i^{th} box, $i = 1, 2, 3$. *(JEE Adv. 2014)*

11. The probability that $x_1 + x_2 + x_3$ is odd, is

(a) $\frac{29}{105}$ (b) $\frac{53}{105}$ (c) $\frac{57}{105}$ (d) $\frac{1}{2}$

12. The probability that x_1, x_2, x_3 are in an arithmetic progression, is

(a) $\frac{9}{105}$ (b) $\frac{10}{105}$ (c) $\frac{11}{105}$ (d) $\frac{7}{105}$

PASSAGE - 6

Let n_1 and n_2 be the number of red and black balls, respectively, in box I. Let n_3 and n_4 be the number of red and black balls, respectively, in box II. *(JEE Adv. 2015)*

13. One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is(are)

(a) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$
(b) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$
(c) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$
(d) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

14. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1 and n_2 is(are)

(a) $n_1 = 4$ and $n_2 = 6$
(b) $n_1 = 2$ and $n_2 = 3$
(c) $n_1 = 10$ and $n_2 = 20$
(d) $n_1 = 3$ and $n_2 = 6$

PASSAGE - 7

Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$ respectively. Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2 respectively after two games.

15. $P(X > Y)$ is *(JEE Adv. 2016)*

(a) $\frac{1}{4}$ (b) $\frac{5}{12}$ (c) $\frac{1}{2}$ (d) $\frac{7}{12}$

16. $P(X = Y)$ is *(JEE Adv. 2016)*

(a) $\frac{11}{36}$ (b) $\frac{1}{3}$ (c) $\frac{13}{36}$ (d) $\frac{1}{2}$

H Assertion & Reason Type Questions

1. Let $H_1, H_2, \ldots, H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, $i = 1, 2, \ldots, n$. Let E be any other event with $0 < P(E) < 1$.

STATEMENT-1 :
$P(H_i | E) > P(E | H_i). P(H_i)$ for $i = 1, 2, \ldots, n$ **because**

STATEMENT-2 : $\sum_{i=1}^{n} P(H_i) = 1$. *(2007 -3 marks)*

(a) Statement-1 is True, statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True.

2. Consider the system of equations $ax + by = 0$; $cx + dy = 0$, where $a, b, c, d \in \{0, 1\}$

STATEMENT - 1 : The probability that the system of equations has a unique solution is $\frac{3}{8}$.

and

STATEMENT - 2 : The probability that the system of equations has a solution is 1. *(2008)*

(a) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1
(b) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 **is NOT** a correct explaination for STATEMENT - 1
(c) STATEMENT - 1 is True, STATEMENT - 2 is False
(d) STATEMENT - 1 is False, STATEMENT - 2 is True

I Integer Value Correct Type

1. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha\ 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1). *(JEE Adv. 2013)*

Then $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$

2. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is *(JEE Adv. 2015)*

Section-B JEE Main / AIEEE

1. A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. Probability that the problem is solved is **[2002]**

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

2. A and B are events such that $P(A \cup B)=3/4$, $P(A \cap B)=1/4$, $P(\overline{A})=2/3$ then $P(\overline{A} \cap B)$ is **[2002]**

(a) 5/12 (b) 3/8 (c) 5/8 (d) 1/4

3. A dice is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is **[2002]**

(a) 8/3 (b) 3/8 (c) 4/5 (d) 5/4

4. The mean and variance of a random variable X having binomial distribution are 4 and 2 respectively, then $P(X=1)$ is **[2003]**

(a) $\frac{1}{4}$ (b) $\frac{1}{32}$ (c) $\frac{1}{16}$ (d) $\frac{1}{8}$

5. Events A, B, C are mutually exclusive events such that $P(A)=\frac{3x+1}{3}$, $P(B)=\frac{1-x}{4}$ and $P(C)=\frac{1-2x}{2}$ The set of possible values of x are in the interval. **[2003]**

(a) $[0, 1]$ (b) $\left[\frac{1}{3}, \frac{1}{2}\right]$ (c) $\left[\frac{1}{3}, \frac{2}{3}\right]$ (d) $\left[\frac{1}{3}, \frac{13}{3}\right]$

6. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is **[2003]**

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{5}$ (d) $\frac{1}{5}$

7. The probability that A speaks truth is $\frac{4}{5}$, while the probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact is **[2004]**

(a) $\frac{4}{5}$ (b) $\frac{1}{5}$ (c) $\frac{7}{20}$ (d) $\frac{3}{20}$

8. A random variable X has the probability distribution:

X:	1	2	3	4	5	6	7	8
p(X):	0.2	0.2	0.1	0.1	0.2	0.1	0.1	0.1

For the events $E=\{X$ is a prime number$\}$ and $F=\{X<4\}$, the $P(E \cup F)$ is **[2004]**

(a) 0.50 (b) 0.77 (c) 0.35 (d) 0.87

9. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is **[2004]**

(a) $\frac{28}{256}$ (b) $\frac{219}{256}$ (c) $\frac{128}{256}$ (d) $\frac{37}{256}$

10. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is **[2005]**

(a) $\frac{2}{9}$ (b) $\frac{1}{9}$ (c) $\frac{8}{9}$ (d) $\frac{7}{9}$

11. A random variable X has Poisson distribution with mean 2. Then $P(X>1.5)$ equals **[2005]**

(a) $\frac{2}{e^2}$ (b) 0 (c) $1-\frac{3}{e^2}$ (d) $\frac{3}{e^2}$

12. Let A and B be two events such that $P(\overline{A \cup B})=\frac{1}{6}$, $P(A \cap B)=\frac{1}{4}$ and $P(\overline{A})=\frac{1}{4}$, where $\overline{A}$ stands for complement of event A. Then events A and B are **[2005]**

(a) equally likely and mutually exclusive
(b) equally likely but not independent
(c) independent but not equally likely
(d) mutually exclusive and independent

13. At a telephone enquiry system the number of phone cells regarding relevant enquiry follow Poisson distribution with an average of 5 phone calls during 10 minute time intervals. The probability that there is at the most one phone call during a 10-minute time period is **[2006]**

(a) $\frac{6}{5^e}$ (b) $\frac{5}{6}$ (c) $\frac{6}{55}$ (d) $\frac{6}{e^5}$

14. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is **[2007]**

(a) 0.2 (b) 0.7 (c) 0.06 (d) 0.14.

15. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is **[2007]**

(a) 8/729 (b) 8/243 (c) 1/729 (d) 8/9.

16. It is given that the events A and B are such that $P(A)=\frac{1}{4}, P(A \mid B)=\frac{1}{2}$ and $P(B \mid A)=\frac{2}{3}$. Then $P(B)$ is **[2008]**

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

17. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is **[2008]**

(a) $\frac{3}{5}$ (b) 0

(c) 1 (d) $\frac{2}{5}$

18. In a binomial distribution $B\left(n, p=\frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than: **[2009]**

(a) $\frac{1}{\log_{10} 4+\log_{10} 3}$ (b) $\frac{9}{\log_{10} 4-\log_{10} 3}$

(c) $\frac{4}{\log_{10} 4-\log_{10} 3}$ (d) $\frac{1}{\log_{10} 4-\log_{10} 3}$

19. One ticket is selected at random from 50 tickets numbered 00,01,02,...,49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals: **[2009]**

(a) $\frac{1}{7}$ (b) $\frac{5}{14}$

(c) $\frac{1}{50}$ (d) $\frac{1}{14}$

20. Four numbers are chosen at random (without replacement) from the set $\{1, 2, 3, ...20\}$. **[2010]**

Statement -1: The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement -2 : If the four chosen numbers form an AP, then the set of all possible values of common difference is $(\pm1, \pm2, \pm3, \pm4, \pm5)$.

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1
(b) Statement -1 is true, Statment -2 is false
(c) Statement -1 is false, Statment -2 is true.
(d) Statement -1 is true, Statement -2 is true ; Statement -2 is a correct explanation for Statement -1.

21. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is **[2010]**

(a) $\frac{2}{7}$ (b) $\frac{1}{21}$

(c) $\frac{2}{23}$ (d) $\frac{1}{3}$

22. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval **[2011]**

(a) $\left(\frac{3}{4}, \frac{11}{12}\right]$ (b) $\left[0, \frac{1}{2}\right]$

(c) $\left(\frac{11}{12}, 1\right]$ (d) $\left(\frac{1}{2}, \frac{3}{4}\right]$

23. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is **[2011]**

(a) $P(C\,|\,D) \geq P(C)$ (b) $P(C\,|\,D) < P(C)$

(c) $P(C\,|\,D) = \frac{P(D)}{P(C)}$ (d) $P(C\,|\,D) = P(C)$

24. Three numbers are chosen at random without replacement from $\{1,2,3,..8\}$. The probability that their minimum is 3, given that their maximum is 6, is : **[2012]**

(a) $\frac{3}{8}$ (b) $\frac{1}{5}$ (c) $\frac{1}{4}$ (d) $\frac{2}{5}$

25. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is: **[JEE M 2013]**

(a) $\frac{17}{3^5}$ (b) $\frac{13}{3^5}$

(c) $\frac{11}{3^5}$ (d) $\frac{10}{3^5}$

26. Let A and B be two events such that $P\left(\overline{A \cup B}\right)=\frac{1}{6}$, $P\left(\overline{A \cap B}\right)=\frac{1}{4}$ and $P\left(\overline{A}\right)=\frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are **[JEE M 2014]**

(a) independent but not equally likely.
(b) independent and equally likely.
(c) mutually exclusive and independent.
(d) equally likely but not independent.

27. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is : **[JEE M 2015]**

(a) $220\left(\frac{1}{3}\right)^{12}$ (b) $22\left(\frac{1}{3}\right)^{11}$

(c) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (d) $55\left(\frac{2}{3}\right)^{10}$

28. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true ? **[JEE M 2016]**

(a) E_1 and E_3 are independent.

(b) E_1, E_2 and E_3 are independent.

(c) E_1 and E_2 are independent.

(d) E_2 and E_3 are independent.

21 Probability

Section-A : JEE Advanced/ IIT-JEE

A 1. $\frac{5}{21}$ 2. $P(A) = P(B)$ 3. 1/9 4. $\frac{1}{3} \le p \le \frac{1}{2}$ 5. 32/55
6. 2/5 7. 5/7 8. 11/16 9. 1/36 10. 1/4

B 1. F 2. F

C 1. (d) 2. (a) 3. (b) 4. (c) 5. (c) 6. (b) 7. (c) 8. (d) 9. (b)
10. (a) 11. (a) 12. (b) 13. (c) 14. (a) 15. (a) 16. (d) 17. (a) 18. (d)
19. (a) 20. (c) 21. (c) 22. (d) 23. (c) 24. (c) 25. (a) 26. (a) 27. (c)

D 1. (a,c,d) 2. (c) 3. (c) 4. (a, b, c) 5. (b, c, d) 6. (a, c) 7. (a, d) 8. (c, d) 9. (a)
10. (a, d) 11. (b) 12. (d) 13. (a) 14. (b) 15. (b, c) 16. (a, d) 17. (b, d) 18. (a, b)
19. (a)

E 1. $\frac{1}{1260}$ 2. (i) $\frac{1}{132}$ (ii) $\frac{1}{462}$ 3. 0.6976 4. No

7. 13.9% 8. 1/5 9. 99/1900 10. 0.37 11. $1 - \frac{10(N+2)}{^{N+7}C_5}$

12. best of three games 13. $\left(\frac{3}{4}\right)^n$ 14. 24/29

15. A, B, C are pairwise independent but A, B, C are dependent. 16. $\frac{97}{(25)^4}$ 17. 0.2436

18. 7(13)!, 12!, $\frac{1}{91}$ 19. 0.62 20. $\alpha = \frac{p}{1-(1-p)^3}, \beta = \frac{(1-p)p}{1-(1-p)^3}, \gamma = \frac{p(1-p)^2}{1-(1-p)^3}$

21. $\frac{4}{35}$ 23. $\frac{m}{m+n}$ 24. $\frac{^6C_3[3^n - 3(2^n) + 3]}{6^n}$ 25. $\frac{9m}{m+8N}$ 26. $2p^2 - p^3$

27. $\frac{1}{2}$ 29. $\frac{^{10}C_1 \times {}^2C_1}{^{12}C_2} \times \frac{^{12}C_2 \times {}^6C_4}{^{18}C_6} + \frac{^{11}C_1 \times {}^1C_1}{^{12}C_2} \times \frac{^{12}C_1 \times {}^6C_5}{^{18}C_6}$ 30. $\frac{1}{7}$

G 1. (b) 2. (a) 3. (b) 4. (a) 5. (b) 6. (d) 7. (b) 8. (d) 9. (a) 10. (d)
11. (b) 12. (c) 13. (a,b) 14. (c,d) 15. (b) 16.(c)

H 1. (d) 2. (b)

I 1. 6 2. 8

Section-B : JEE Main/ AIEEE

1. (a) 2. (a) 3. (d) 4. (b) 5. (b) 6. (a) 7. (c) 8. (b) 9. (a) 10. (b)
11. (c) 12. (c) 13. (d) 14. (d) 15. (b) 16. (b) 17. (c) 18. (d) 19. (d) 20. (b)
21. (a) 22. (b) 23. (a) 24. (b) 25. (c) 26. (a) 27. (c) 28. (b)

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. Let $E_1 \equiv$ face 1 has turned up, $E_2 \equiv$ face 1 or 2 has turned up.
By the given data
$P(E_2)=0.1+0.32=0.42, \quad P(E_1 \cap E_2)=P(E_1)=0.1$
Given that E_2 has happened and we have to find the probability of happening of E_1.
$\therefore$ By conditional probability theorem, we have

$$P(E_1/E_2)=\frac{P(E_1 \cap E_2)}{P(E_2)}=\frac{0.1}{0.42}=\frac{10}{42}=\frac{5}{21}.$$

2. Given that $P(A \cup B)=P(A \cap B)$
$\Rightarrow P(A)+P(B)-P(A \cap B)=P(A \cap B)$
$\Rightarrow [P(A)-P(A \cap B)]+[P(B)-P(A \cap B)]=0$
But $P(A)-P(A \cap B), P(B)-P(A \cap B) \geq 0$
$[\because P(A \cap B) \leq P(A), P(B)]$
$\Rightarrow P(A)-P(A \cap B)=0$ and $P(B)-P(A \cap B)=0$
[$\because$ Sum of two non-negative no's can be zero only when these no's are zeros]
$\Rightarrow P(A)=P(B)=P(A \cap B)$
which is the required relationship.

3. Let A be the event that max. number on the two chosen tickets in not more than 10, and B is the event that min. number on them is 5. We have to find $P(B/A)$.

We know that $P(B/A)=\dfrac{P(B \cap A)}{P(A)}$

Total ways to select two tickets out of 100 = ${}^{100}C_2$.
Number of ways favourable to A
= number of ways of selecting any 2 numbers from 1 to 10
$= {}^{10}C_2=45$
$A \cap B$ contains one number 5 and other greater than 5 and ≤ 10
So ways favourable to $A \cap B = {}^5C_1=5$

Therefore, $P(A)=\dfrac{45}{{}^{100}C_2}$ and $P(B \cap A)=\dfrac{5}{{}^{100}C_2}$

Thus, $P(B/A)=\dfrac{5/{}^{100}C_2}{45/{}^{100}C_2}=\dfrac{5}{45}=\dfrac{1}{9}$

4. Let $P(A)=\dfrac{1+3p}{3}, P(B)=\dfrac{1-p}{4}, P(C)=\dfrac{1-2p}{2}$
As A, B and C are three mutually exclusive events
$\therefore P(A)+P(B)+P(C) \leq 1$

$$\Rightarrow \frac{1+3p}{3}+\frac{1-p}{4}+\frac{1-2p}{2} \leq 1$$

$\Rightarrow 4+12p+3-3p+6-12p \leq 12$
$\Rightarrow 3p \geq 1 \Rightarrow p \geq 1/3$ ...(i)

Also $0 \leq P(A) \leq 1 \Rightarrow 0 \leq \dfrac{1+3p}{3} \leq 1$

$\Rightarrow 0 \leq 1+3p \leq 3$

$\Rightarrow -\dfrac{1}{3} \leq p \leq \dfrac{2}{3}$ ...(ii)

$0 \leq P(B) \leq 1 \Rightarrow 0 \leq \dfrac{1-p}{4} \leq 1$

$\Rightarrow 0 \leq 1-p \leq 4$
$\Rightarrow -3 \leq p \leq 1$ ...(iii)

$0 \leq P(C) \leq 1 \Rightarrow 0 \leq \dfrac{1-2p}{2} \leq 1 \Rightarrow -\dfrac{1}{2} \leq p \leq \dfrac{1}{2}$ ...(iv)

Combining (i), (ii), (iii) and (iv), we get $\dfrac{1}{3} \leq p \leq \dfrac{1}{2}$

5. There may be following cases:
Case I : Red from A to B and red from B to A then prob. of drawing a red ball from A $=\dfrac{6}{10}\times\dfrac{5}{11}\times\dfrac{6}{10}=\dfrac{180}{1100}=\dfrac{18}{110}$
Case II : Red from A to B and black from B to A then prob. of drawing a red from A $=\dfrac{6}{10}\times\dfrac{6}{11}\times\dfrac{5}{10}=\dfrac{180}{1100}=\dfrac{18}{110}$
Case III : Black from A to B and red from B to A then prob. of drawing red from A $=\dfrac{4}{10}\times\dfrac{4}{11}\times\dfrac{7}{10}=\dfrac{56}{550}$
Case IV : Black from A to B and black from B to A then prob. of drawing red from A $=\dfrac{4}{10}\times\dfrac{7}{11}\times\dfrac{6}{10}=\dfrac{168}{1100}=\dfrac{84}{550}$

$\therefore$ The required prob $=\dfrac{18}{110}+\dfrac{18}{110}+\dfrac{56}{550}+\dfrac{84}{550}$

$$=\frac{90+90+56+84}{550}=\frac{320}{550}=\frac{32}{55}$$

6. Probability of getting a sum of $5=\dfrac{4}{36}=\dfrac{1}{9}=P(A)$ as favourable cases are $\{(1,4),(4,1),(2,3),(3,2)\}$
Similarly favourable cases of getting a sum of 7 are $\{(1,6),(6,1),(2,5),(5,2),(3,4),(4,3)\}=6$

$\therefore$ Prob. of getting a sum of $7=\dfrac{6}{36}=\dfrac{1}{6}$

$\therefore$ Prob. of getting a sum of 5 or 7

$=\dfrac{1}{6}+\dfrac{1}{9}=\dfrac{5}{18}$ [as events are mutually exclusive.]

$\therefore$ Prob of getting neither a sum of 5 nor of 7 $=\dfrac{1}{6}-\dfrac{5}{18}=\dfrac{13}{18}$

Now we get a sum of 5 before a sum of 7 if either we get a sum of 5 in first chance or we get neither a sum of 5 nor of 7 in first chance and a sum of 5 in second chance and so on.
Therefore the required prob. is

$$=\frac{1}{9}+\frac{13}{18}\times\frac{1}{9}+\frac{13}{18}\times\frac{13}{18}\times\frac{1}{9}+\ldots\infty=\frac{1/9}{1-13/18}=\frac{1}{9}\times\frac{18}{5}=\frac{2}{5}$$

7. $P(A \cup B)=0.8$
$\Rightarrow P(A \cup B)=P(A)+P(B)-P(A \cap B)$

$\Rightarrow\ P(A\cup B)=P(A)+P(B)-P(A)P(B)$
[As A and B are independent events]
$\Rightarrow\ 0.8=0.3+P(B)-0.3P(B)$
$\Rightarrow\ 0.5=0.7P(B)\Rightarrow P(B)=5/7$

8. For a binomial distribution, we know,
mean = np and variance = npq
$\therefore\ np=2;\ npq=1\Rightarrow q=1/2$
$\Rightarrow\ p=1/2$ and $n=4$
$\therefore\ P(X>1)=P(X=2)+P(X=3)+P(X=4)$
$=1-P(X=0)-P(X=1)$

$$=1-{}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4-{}^4C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3=1-\frac{1}{16}-\frac{4}{16}=\frac{11}{16}$$

9. Sample space = {Y, Y, Y, R, R, B} where Y stands for yellow colour, R for red and B for blue.
Prob. that the colours yellow, red and blue appear in the first second, and third tosses respectively $=\frac{3}{6}\times\frac{2}{6}\times\frac{1}{6}=\frac{1}{36}$

10. Given that $P(A^c)=0.3, P(B)=0.4$ and $P(A\cap B^c)=0.5$

then $P[B/(A\cap B^c)]=\dfrac{P[B\cap(A\cup B^c)]}{P(A\cup B^c)}$

$$=\frac{P((B\cap A)\cup(B\cap B^c))}{P(A\cup B^c)}=\frac{P(A\cap B)}{P(A)+P(B^c)-P(A\cap B^c)}$$

$$=\frac{P(A)-P(A\cap B^c)}{1-P(A^c)+1-P(B)-P(A\cap B^c)}$$

$$=\frac{1-0.3-0.5}{1-0.3+1-0.4-0.5}=\frac{0.2}{0.8}=\frac{1}{4}$$

B. True / False

1. Let E be the event "No two S's occur together".

A, A, I, N can be arranged in $\frac{4!}{2!}=12$ ways

$-A-A-I-N-$ Creating 5 places for 4 S. Out of 5 places 4 can be selected in ${}^5C_4=5$ ways.

$\therefore$ No two S's occur together in $=12\times5=60$ ways
Total no.of arranging all letters of word 'assassin'
$=\frac{8!}{4!2!}=840$

$\therefore$ Req. prob. $=\frac{60}{840}=\frac{1}{14}$ $\therefore$ Statement is False.

2. $P(A\cup B)=P(A)+P(B)-P(A\cap B)$
$=P(A)+P(B)-P(A)P(B)$
[$\because$ A and B are independent events]
$=0.2+0.3-0.2\times0.3=0.5-0.06=0.44\neq0.5$
$\therefore$ The statement is false.

C. MCQs with ONE Correct Answer

1. **(d)** The two events can happen simultaneously e.g., (2, 3)
$\therefore$ not mutually exclusive.
Also are not dependent on each other.

2. **(a)** $P(A\cup B)=P(A)+P(B)-P(A\cap B)$
$=0.25+0.50-0.14=0.61$
$\therefore\ P(A'\cap B')=P((A\cup B)')=1-P(A\cup B)$
$=1-0.61=0.39$

3. **(b)** $p=0.4, n=3, P(X\geq1)=?\Rightarrow q=0.6$
$\therefore\ P(X\geq1)=1-P(X=0)$
$=1-{}^3C_0(0.4)^0(0.6)^3=1-0.216=0.784$

4. **(c)** $P(\bar{A}/\bar{B})=\dfrac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}=\dfrac{P(\overline{A\cup B})}{P(\bar{B})}=\dfrac{1-P(A\cup B)}{P(\bar{B})}$

5. **(c)** $n=7$
Prob. of getting any no. out 1, 2, 3, ... 9 is $p=9/15$
$\therefore\ q=6/5$
$P(x=7)={}^7C_7p^7q^0$ [Binomial distribution]
$$=\left(\frac{9}{15}\right)^7=\left(\frac{3}{5}\right)^7$$

6. **(b)** Favourable cases = 6; {(1, 1, 1), (2, 2, 2),...(6, 6, 6)}

Total cases $=6\times6\times6=216$ $\therefore$ Req. prob. $=\frac{6}{216}=\frac{1}{36}$

7. **(c)** Prob. of a getting a white ball in a single draw
$=p=\frac{12}{24}=\frac{1}{2}$
Prob. of getting a white ball 4th time in the 7th draw
$=P$ (getting 3 W in 6 draws)
$\times P$ (getting W ball at 7th draw)

$$={}^6C_3\left(\frac{1}{2}\right)^6\cdot\frac{1}{2}=\frac{5}{32}$$

8. **(d)** Prob. of one coin showing head = p
$\therefore$ Prob of one coin showing tail $=1-p$
ATQ coin is tossed 100 times and prob. of 50 coins showing head = prob of 51 coins showing head.
Using binomial prob. distribution

$P(X=r)={}^nC_rp^rq^{n-r}$,
we get, ${}^{100}C_{50}p^{50}(1-p)^{50}={}^{100}C_{51}p^{51}(1-p)^{49}$

$$\Rightarrow\ \frac{1-p}{p}=\frac{{}^{100}C_{51}}{{}^{100}C_{50}}=\frac{50!50!}{51!49!}=\frac{50}{51}\Rightarrow 51-51p=50p$$

$$\Rightarrow\ 101p=51\Rightarrow p=\frac{51}{101}$$

9. **(b)** P (at least 7 pts) $=P$ (7pts) $+P$ (8 pts)
[$\because$ At most 8 pts can be scored.]
Now 7 pts can be scored by scoring 2 pts in 3 matches and 1 pt. in one match, similarly 8 pts can be scored by scoring 2 pts in each of the 4 matches.
$\therefore$ Req. prob. $={}^4C_1\times[P(2\text{ pts})]^3P(1\text{pt})+[P(2\text{ pts})]^4$
$=4(0.5)^3\times0.05+(0.50)^4=0.0250+0.0625=0.0875$

10. **(a)** The min. face value is not less than 2 and max. face value is not greater than 5 if we get any of the numbers 2, 3, 4, 5, while total possible out comes are 1, 2, 3, 4, 5 and 6.
$\therefore$ In one thrown of die, prob. of getting any no.

Out of 2, 3, 4 and 5 is $=\frac{4}{6}=\frac{2}{3}$
If the die is rolled four times, then all these events being independent, the required prob. $\left(\frac{2}{3}\right)^4=\frac{16}{81}$

11. **(a)** $P[A\cap(B\cup C)]=P[(A\cap B)\cup(A\cap C)]$
$=P(A\cap B)+P(A\cap C)-P(A\cap B\cap C)$
$=P(A)\ P(B)+P(A)\ P(C)-P(A)\ P(B)P(C)$
$=P(A)[P(B)+P(C)-P(B\cap C)]=P(A)P(B\cup C)$

$\therefore$ S_1 is true.

$P(A \cap (B \cap C)) = P(A)P(B)P(C) = P(A)P(B \cap C)$

$\therefore$ S_2 is also true.

12. (b) Given that P (India wins) $= p = 1/2$

$\therefore$ P (India loses) $= p' = 1/2$

Out of 5 matches india's second win occurs at third test

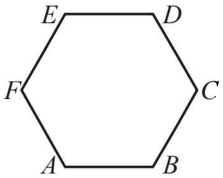
$\Rightarrow$ India wins third test and simultaneously it has won one match from first two and lost the other.

$\therefore$ Required prob. $= P(LWW) + P(WLW)$

$$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 = \frac{1}{4}$$

13. (c) Out of 6 vertices 3 can be chosen in ${}^6C_3$ ways.

Δ will be equilateral if it is ΔACE or ΔBDF (2 ways)



$\therefore$ Required prob. $= \frac{2}{{}^6C_3} = \frac{2}{20} = \frac{1}{10}$

14. (a) We know that P (exactly one of A or B occurs)

$= P(A) + P(B) - 2P(A \cap B)$.

Therefore, $P(A) + P(B) - 2P(A \cap B) = p$ ...(1)

Similarly, $P(B) + P(C) - 2P(B \cap C) = p$ ...(2)

and $P(C) + P(A) - 2P(C \cap A) = p$ ...(3)

Adding (1), (2) and (3) we get

$2[P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)] = 3p$

$\Rightarrow P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = 3p/2$ ...(4)

We are also given that,

$P(A \cap B \cap C) = p^2$ ...(5)

Now, P (at least one of A, B and C)

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{3p}{2} + p^2$ [using (4) and (5)] $= \frac{3p + 2p^2}{2}$

15. (a) We know that,

$7^1 = 7, 7^2 = 49, 7^3 = 343, 7^4 = 2401, 7^5 = 16807$

$\therefore$ 7^k (where $k \in Z$), results in a number whose unit's digit is 7 or 9 or 3 or 1.

Now, $7^m + 7^n$ will be divisible by 5 if unit's place digit of resulting number is 5 or 0 clearly it can never be 5. But it can be 0 if we consider values of m and n such that the sum of unit's place digits become 0. And this can be done by choosing

$\left.\begin{array}{l} m = 1, 5, 9, \ldots 97 \\ \text{and correspondingly} \\ n = 3, 7, 11, \ldots 99 \end{array}\right\}$ (25 options each) $[7 + 3 = 10]$

$\left.\begin{array}{l} m = 2, 6, 10, \ldots\ldots 98 \\ \text{and} \\ n = 4, 8, 12, \ldots\ldots 100 \end{array}\right\}$ (25 options each) $[9 + 1 = 10]$

Case I : Thus m can be chosen in 25 ways and n can be chosen in 25 ways

Case II : m can be chosen in 25 ways and n can be chosen in 25 ways

$\therefore$ Total no. of selections of m, n so that $7^m + 7^n$ is divisible by $5 = (25 \times 25 + 25 \times 25) \times 2$

Note we can interchange values of m and n.

Also no. of total possible selections of m and n out of $100 = 100 \times 100$

$\therefore$ Req. prob. $= \frac{2(25 \times 25 + 25 \times 25)}{100 \times 100} = \frac{1}{4}$.

16. (d) The minimum of two numbers will be less than 4 if at least one of the numbers is less than 4.

$\therefore$ P (at least one no. is < 4),

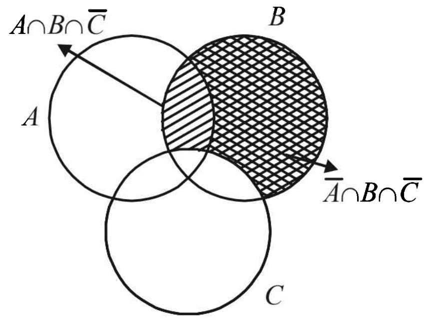
$= 1 - P$ (both the no's are ≥ 4)

$= 1 - \frac{3}{6} \times \frac{2}{5} = 1 - \frac{6}{30} = 1 - \frac{1}{5} = 4/5$

17. (a) Given that $P(B) = 3/4$, $P(A \cap B \cap \bar{C}) = 1/3$

$P(\bar{A} \cap B \cap \bar{C}) = 1/3$

From venn diagram, we see



$B \cap C \equiv B - (A \cap B \cap \bar{C}) - (\bar{A} \cap B \cap \bar{C})$

$\Rightarrow P(B \cap C) = P(B) - P(A \cap B \cap \bar{C}) - P(\bar{A} \cap B \cap \bar{C})$

$\Rightarrow P(B \cap C) = \frac{3}{4} - \frac{1}{3} - \frac{1}{3} = \frac{9 - 4 - 4}{12} = \frac{1}{12}$

18. (d) If a no. is to be divisible by both 2 and 3. It should be divisible by their L.C.M.

$\therefore$ L.C.M. of (2 and 3) = 6

$\therefore$ Numbers are $= 6, 12, 18 \ldots 96$.

Total numbers are = 16

$\therefore$ Probability $= \frac{{}^{16}C_3}{{}^{100}C_3} = \frac{4}{1155}$

19. (a) In single throw of a dice, probability of getting 1 is $= \frac{1}{6}$

and prob. of not getting 1 is $\frac{5}{6}$.

Then getting 1 in even no. of chances = getting 1 in 2nd chance or in 4th chance or in 6th chance and so on

$\therefore$ Req. Prob. $= \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^3 \times \frac{1}{6} + \left(\frac{5}{6}\right)^5 \times \frac{1}{6} + \ldots\infty$

$$= \frac{5}{36}\left[\frac{1}{1 - \frac{25}{36}}\right] = \frac{5}{36} \times \frac{36}{11} = \frac{5}{11}$$

20. (c) Let $E_1 \equiv$ The Indian man is seated adjacent to his wife.
$E_2 \equiv$ Each American man is seated adjacent to his wife.

Then $P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$

Now $E_1 \cap E_2 \equiv$ All men are seated adjacent to their wives.

$\therefore$ We can consider the 5 couples as single-single objects which can be arranged in a circle in 4! ways. But for each couple, husband and wife can interchange their places in 2! ways.

$\therefore$ Number of ways when all men are seated adjacent to their wives $= 4! \times (2!)^5$

Also in all 10 persons can be seated in a circle in 9! ways.

$\therefore \quad P(E_1 \cap E_2) = \frac{4! \times (2!)^5}{9!}$

Similarly if each American man is seated adjacent to his wife, considering each American couple as single object and Indian woman and man as seperate objects there are 6 different objects which can be arranged in a circle in 5! ways. Also for each American couple, husband and wife can interchange their places in 2 ! ways.

So the number of ways in which each American man is seated adjacent to his wife.

$= 5! \times (2!)^4 \therefore P(E_2) = \frac{5! \times (2!)^4}{9!}$

So $P(E_1/E_2) = \frac{(4! \times (2!)^5)/9!}{(5! \times (2!)^4)/9!} = \frac{2}{5}$

21. (c) $P(E^c \cap F^c / G) = P(E \cup F)^c / G)$

$1 - P(E \cup F / G)$

$= 1 - P(E/G) - P(F/G) + P(E \cap F/G)$

$= 1 - P(E) - P(F) + O$

($\because$ E, F, G are pairwise independent and $P(E \cap F \cap G) = 0$

$\Rightarrow P(E).P(F) = 0$ as $P(G) > 0$) $= P(E^c) - P(F)$

22. (d) We have n (S) = 10, n(A) = 4

Let $n(B) = x$ and $n(A \cap B) = y$

Then for A and B to be independent events

$P(A \cap B) = P(A) P(B)$

$\Rightarrow \frac{y}{10} = \frac{4}{10} \times \frac{x}{10} \Rightarrow x = \frac{5}{2} y$

$\Rightarrow$ y can be 2 or 4 so that $x = 5$ or 10

$\therefore n(B) = 5$ or 10

23. (c) If ω is a complex cube root of unity then, we know that

$\omega^{3m} + \omega^{3n+1} + \omega^{3p+2} = 0$

where m, n, p are integers.

$\therefore$ r_1, r_2, r_3 should be of the form $3m$, $3n + 1$ and $3p + 2$ taken in any order. As r_1, r_2, r_3 are the numbers obtained on die, these can take any value from 1 to 6.

$\therefore$ m can take values 1 or 2, n can take values 0 or 1

p can take values 0 or 1

$\therefore$ Number of ways of selecting r_1, r_2, r_3

$= {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times 3!$.

Also the total number of ways of getting r_1, r_2, r_3 on die $= 6 \times 6 \times 6$

$\therefore$ Required probability $= \frac{{}^2C_1 \times {}^2C_1 \times {}^2C_1 \times 3!}{6 \times 6 \times 6} = \frac{2}{9}$

24. (c) Let G ≡ oreginal signal is green ⇒ P(G) = 4/5

$E_1 \equiv$ A receives the signal correctly $P(E_1) = 3/4$

$E_2 \equiv$ B receives the signal correctly $P(E_2) = 3/4$

E ≡ Signal received by B is green.

Then E can happen in the following ways

Original Signal	Received at A	Received at B
Red ⟶	Red ⟶	Green
Red ⟶	Green ⟶	Green
Green ⟶	Green ⟶	Green
Green ⟶	Red ⟶	Green

$\therefore$ $P(E) =)(\bar{G} \cap E_1 \cap \bar{E}_2) + (\bar{G} \cap E_1 \cap \bar{E}_2)$

$+ P(G \cap E_1 \cap E_2) + P(G \cap \bar{E}_1 \cap \bar{E}_2)$

$= \frac{1}{5} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} \times \frac{1}{4} + \frac{4}{5} \times \frac{3}{4} \times \frac{3}{4} + \frac{4}{5} \times \frac{1}{4} \times \frac{1}{4}$

$= \frac{3+3+36+4}{80} = \frac{46}{80} = \frac{23}{40}$

$\therefore P(G/E) = \frac{P(G \cap E)}{P(E)} = \frac{P(G \cap E_1 \cap E_2) + P(G \cap \bar{E}_1 \cap \bar{E}_2)}{P(E)}$

$= \frac{\frac{4}{5} \times \frac{3}{4} \times \frac{3}{4} + \frac{4}{5} \times \frac{1}{4} \times \frac{1}{4}}{23/40} = \frac{40/80}{23/40} = \frac{20}{23}$

25. (a) D_4 can show a number appearing on one of D_1, D_2 and D_3 in the following cases.

Case I : D_4 shows a number which is shown by only one of D_1, D_2 and D_3.

D_4 shows a number in ${}^6C_1$ ways.

One out of D_1, D_2 and D_3 can be selected in ${}^3C_1$ ways.

The selected die shows the same number as on D_4 in one way and rest two dice show the different number in 5 ways each.

$\therefore$ Number of ways to happen case I

$= {}^6C_1 \times {}^3C_1 \times 1 \times 5 \times 5 = 450$

Case II : D_4 shows a number which is shown by only two of D_1, D_2 and D_3.

As discussed in case I, it can happen in the following number of ways

$= {}^6C_1 \times {}^3C_2 \times 1 \times 1 \times 5 = 90$

Case III : D_4 shows a number which is shown by all three dice D_1, D_2 and D_3.

Number of ways it can be done

$= {}^6C_1 \times {}^3C_3 \times 1 \times 1 \times 1 = 6$

$\therefore$ Total number of favourable ways $= 450 + 90 + 6 = 546$

Also total ways $= 6 \times 6 \times 6 \times 6$

$\therefore$ Required Probability $= \frac{546}{6 \times 6 \times 6 \times 6} = \frac{91}{216}$

26. (a) According to given condition, we can have the following cases

(I) GGBBB (II) BGGBB
(III) GBGBB (IV) BGBGB
(V) GBBGB

i.e., the two girls can occupy two of the first three places (case I, II, III) or second and fourth (case IV) or first and fourth (case V) places.

Thus favourable cases are $= 3 \times 2! \times 3! + 2 \times 2! \times 3! = 60$

Total ways in which 5 persons can be seated $= 5! = 120$

$\therefore$ Required probability $= \frac{60}{120} = \frac{1}{2}$

27. (c) $P(T_1) = \frac{20}{100}, P(T_2) = \frac{80}{100}, P(D) = \frac{7}{100}$

Let $P\left(\frac{D}{T_2}\right) = x$, then $P\left(\frac{D}{T_1}\right) = 10x$

Also $P(D) = P(T_1)\, P\left(\frac{D}{T_1}\right) + P(T_2) P\left(\frac{D}{T_2}\right)$

$\Rightarrow \frac{7}{100} = \frac{20}{100} \times 10x + \frac{80}{100} \times x$

$\Rightarrow \frac{7}{280} = x$ or $x = \frac{1}{40}$

$P\left(\frac{D}{T_1}\right) = \frac{10}{40}$ and $P\left(\frac{D}{T_2}\right) = \frac{1}{40}$

$\Rightarrow P\left(\frac{\bar{D}}{T_1}\right) = \frac{30}{40}$ and $P\left(\frac{\bar{D}}{T_2}\right) = \frac{39}{40}$

$$P\left(\frac{T_2}{\bar{D}}\right) = \frac{P\left(\frac{\bar{D}}{T_2}\right) P(T_2)}{P\left(\frac{\bar{D}}{T_1}\right) P(T_1) + P\left(\frac{\bar{D}}{T_2}\right) P(T_2)}$$

$$= \frac{\frac{80}{100} \times \frac{39}{40}}{\frac{20}{100} \times \frac{30}{40} + \frac{80}{100} \times \frac{39}{40}} = \frac{156}{186} = \frac{26}{31}$$

Also $\frac{78}{93} = \frac{26}{31}$

D. MCQs with ONE or MORE THAN ONE Correct

1. (a, c, d) Given that M and N are any two events. To check the probability that exactly one of them occurs. We check all the options one by one.

(a) $P(M) + P(N) - 2P(M \cap N)$
$= [P(M) + P(N) - P(M \cap N)] - P(M \cap N)$
$= P(M \cup N) - P(M \cap N)$
$\Rightarrow$ Prob. that exactly one of M and N occurs.

(b) $P(M) + P(N) - P(M \cap N) = P(M \cup N)$
$\Rightarrow$ Prob. that at least one of M and N occurs.

(c) $P(M^c) + P(N^c) - 2P(M^c \cap N^c)$
$= 1 - P(M) + 1 - P(N) - 2P(M \cup N)^c$
$= 2 - P(M) - P(N) - 2[1 - P(M \cup N)]$
$= 2 - P(M) - P(N) - 2 + 2P(M \cup N)$
$= P(M \cup N) + P(M \cup N) - P(M) - P(N)$
$= P(M \cup N) - P(M \cap N)$
$\Rightarrow$ Prob. that exactly one of M and N occurs.

(d) $P(M \cap N^c) + P(M^c \cap N)$
$\Rightarrow$ Prob that M occurs but not N or prob that M does not occur but N occurs.
$\Rightarrow$ Prob. that exactly one of M and N occurs.

Thus we can conclude that (a), (c) and (d) are the correct options.

2. (c) Let A, B, C be the events that the student passes test I, II, III respectively.

Then, ATQ ; $P(A) = p; P(B) = q; P(C) = \frac{1}{2}$

Now the student is successful if A and B happen or A and C happen or A, B and C happen.

ATQ, $P(AB\bar{C}) + P(AC\bar{B}) + P(ABC) = \frac{1}{2}$

$\Rightarrow pq\left(1 - \frac{1}{2}\right) + p.\frac{1}{2}.(1-q) + p.q.\frac{1}{2} = \frac{1}{2}$

$\Rightarrow \frac{1}{2}pq + \frac{1}{2}p - \frac{1}{2}pq + \frac{1}{2}pq = \frac{1}{2}$

$\Rightarrow p + pq = 1 \Rightarrow p(1+q) = 1$

which holds for $p = 1$ and $q = 0$.

3. (c) Given that $P(A \cup B) = 0.6$; $P(A \cap B) = 0.2$

$\therefore P(\bar{A}) + P(\bar{B}) = 1 - P(A) + 1 - P(B)$
$= 2 - (P(A) + P(B)) = 2 - [P(A \cup B) + P(A \cap B)]$.
$= 2 - [0.6 + 0.2] = 2 - 0.8 = 1.2$

4. (a, b, c) We know that,

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$ ...(1)

Also $P(A \cup B) \le 1$

$\Rightarrow -P(A \cup B) \ge -1$ ...(2)

$\therefore P(A \cap B) \ge P(A) + P(B) - 1$ [Using (1) and (2)]

$\therefore$ (a) is true. Again $P(A \cup B) \ge 0$

$\Rightarrow -P(A \cup B) \le 0$ ...(3)

$\Rightarrow P(A \cap B) \le P(A) + P(B)$ [Using (1) and (3)]

$\therefore$ (b) is also correct.

From (1) (c) is true and (d) is not correct.

5. (b, c, d) Since E and F are independent

$\therefore P(E \cap F) = P(E).P(F)$ ...(1)

Now, $P(E \cap F^c) = P(E) - P(E \cap F)$
$= P(E) - P(E)P(F)$ [Using (1)]
$= P(E)[1 - P(F)] = P(E)P(F^c)$

$\therefore$ E and F^c are independent.

Again $P(E^c \cap F^c) = P(E \cup F)^c = 1 - P(E \cup F)$
$= 1 - P(E) - P(F) + P(E \cap F)$
$= 1 - P(E) - P(F) + P(E)P(F)$
$= ((1 - P(E))(1 - P(F)) = P(E^c)P(F^c)$

$\therefore$ E^c and F^c are independent.

Also $P(E/F) + P(E^c/F)$

$$= \frac{P(E \cap F)}{P(F)} + \frac{P(E^c \cap F)}{P(F)} = \frac{P(E)P(F) + P(E^c)P(F)}{P(F)}$$

$$= \frac{P(F)(P(E) + P(E^c))}{P(F)} = 1$$

6. (a, c) For any two events A and B

(a) $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

Now we know $P(A \cup B) \le 1$

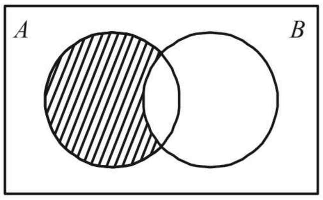
$P(A) + P(B) - P(A \cap B) \le 1$

$\Rightarrow P(A \cap B) \ge P(A) + P(B) - 1$

$\Rightarrow \dfrac{P(A \cap B)}{P(B)} \ge \dfrac{P(A) + P(B) - 1}{P(B)}$ $\left[\begin{array}{l}\text{As } P(B) \ne 0 \\ \therefore P(B) > 0\end{array}\right]$

$\Rightarrow P(A/B) \ge \dfrac{P(A) + P(B) - 1}{P(B)}$ $\therefore$(a) is correct statement.

(b)



From venn diagram we can clearly conclude that

$P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$\therefore$ (b) is incorrect statement.

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 1 - P(\bar{A}) + 1 - P(\bar{B}) - P(A)P(B)$

[$\because$ A & B are independent events]

$= 2 - P(\bar{A}) - P(\bar{B}) - [1 - P(\bar{A})][1 - P(\bar{B})]$

$= 2 - P(\bar{A}) - P(\bar{B}) - 1 + P(\bar{A}) + P(\bar{B}) - P(\bar{A})P(\bar{B})]$

$= 1 - P(\bar{A})P(\bar{B})$ $\therefore$ (c) is the correct statement.

(d) For disjoint events $P(A \cup B) = P(A) + P(B)$

$\therefore$ (d) is the incorrect statement.

7. (a, d) Let $P(E) = x$ and $P(F) = y$

ATQ, $P(E \cap F) = \dfrac{1}{12}$

As E and F are independent events

$\therefore$ $P(E \cap F) = P(E)P(F)$

$\Rightarrow \dfrac{1}{12} = xy \Rightarrow xy = \dfrac{1}{12}$ ...(1)

Also $P(\bar{E} \cap \bar{F}) = P(\overline{E \cup F}) = 1 - P(E \cup F)$

$\Rightarrow \dfrac{1}{2} = 1 - [P(E) + P(F) - P(E)P(F)]$

$\Rightarrow x + y - xy = \dfrac{1}{2} \Rightarrow x + y = \dfrac{7}{12}$ ...(2)

Solving (1) and (2) we get

either $x = \dfrac{1}{3}$ and $y = \dfrac{1}{4}$ or $x = \dfrac{1}{4}$ and $y = \dfrac{1}{3}$

$\therefore$ (a) and (d) are the correct options.

8. (c, d) $P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - P(A) - P(B) + P(A)P(B)$

$= P(A')P(B')$

Also $P(A \cup B) = P(A) + P(B) - P(A)P(B)$

$\Rightarrow P(A \cap B) = P(A) + P(B)$

$\therefore$ $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A)P(B)}{P(B)} = P(A)$

9. (a) P(2 white and 1 black)

$= P(W_1 W_2 B_3 \text{ or } W_1 B_2 W_3 \text{ or } B_1 W_2 W_3)$

$= P(W_1 W_2 B_3) + P(W_1 B_2 W_3) + P(B_1 W_2 W_3)$

$= P(W_1)P(W_2)P(B_3) + P(W_1)P(B_2)P(W_3) + P(B_1)P(W_2)P(W_3)$

$= \dfrac{3}{4}.\dfrac{2}{4}.\dfrac{3}{4} + \dfrac{3}{4}.\dfrac{2}{4}.\dfrac{1}{4} + \dfrac{1}{4}.\dfrac{2}{4}.\dfrac{1}{4} = \dfrac{1}{32}(9 + 3 + 1) = \dfrac{13}{32}$

10. (a, d) We have,

(a) $P(E/F) + P(\bar{E}/F) = \dfrac{P(E \cap F)}{P(F)} + \dfrac{P(\bar{E} \cap F)}{P(F)}$

$= \dfrac{P(E \cap F) + P(\bar{E} \cap F)}{P(F)} = \dfrac{P(F)}{P(F)} = 1$

$\therefore$ (a) holds.

Also

(b) $P(E/F) + P(E/\bar{F}) = \dfrac{P(E \cap F)}{P(F)} + \dfrac{P(E \cap F)}{P(\bar{F})}$

$= \dfrac{P(E \cap F)[1 - P(F)] + P(F)[P(E \cap \bar{F})]}{P(F)P(\bar{F})}$

$= \dfrac{P(E \cap F) + P(F)[P(E \cap \bar{F}) - P(E \cap F)]}{P(F)P(\bar{F})} \ne 1$

$\therefore$ (b) does not hold. Similarly we can show that (c) does not hold but (d) holds.

11. (b) The probability that only two tests are needed = (probability that the second machine tested is faulty given the first machine tested is faulty) $= \dfrac{2}{4} \times \dfrac{1}{3} = \dfrac{1}{6}$

12. (d) Given that $P(E) \le P(F)$ and $P(E \cap F) > 0$. It doesn't necessarily mean that E is the subset of F.

$\therefore$ The choices (a), (b), (c) do not hold in general.

Hence (d) is the right choice here.

13. (a) The event that the fifth toss results in a head is independent of the event that the first four tosses result in tails.

$\therefore$ Probability of the required event = 1/2.

14. (b) The no. of ways of placing 3 black balls without any restriction is ${}^{10}C_3$. Now the no. of ways in which no two black balls put together is equal to the no of ways of choosing 3 places marked out of eight places.

$- W - W - W - W - W - W - W -$

This can be done is ${}^{8}C_3$ ways. Thus, probability of the

required event $= \dfrac{{}^{8}C_3}{{}^{10}C_3} = \dfrac{8 \times 7 \times 6}{10 \times 9 \times 8} = \dfrac{7}{15}$.

$\therefore$ (b) is the correct option.

15. (b, c) According to the problem,

$m + p + c - mp - mc - pc + mpc = 3/4$ ...(1)

$mp(1 - c) + mc(1 - p) + pc(1 - m) = 2/5$

or $mp + mc + pc - 3mpc = 2/5$ ...(2)

Also $mp + pc + mc - 2mpc = 1/2$ ...(3)

(2) and (3) $\Rightarrow mpc = \dfrac{1}{2} - \dfrac{2}{5} = \dfrac{1}{10}$

$\therefore$ $mp + mc + pc = \dfrac{2}{5} + \dfrac{3}{10} = \dfrac{7}{10}$

$\therefore$ $m + p + c = \dfrac{3}{4} + \dfrac{7}{10} - \dfrac{1}{10} = \dfrac{15 + 14 - 2}{20} = \dfrac{27}{20}$

16. **(a,d)** $\because$ E and F are independent events

$\therefore P(E \cap F) = P(E).P(F)$ ...(1)

Given that $P(E \cap \bar{F}) + P(\bar{E} \cap F) = \frac{11}{25}$

$\Rightarrow P(E)P(\bar{F}) + P(\bar{E})P(F) = \frac{11}{25}$

$\Rightarrow P(E)(1 - P(F)) + (1 - P(E))P(F) = \frac{11}{25}$

$\Rightarrow P(E) - P(E)P(F) + P(F) - P(E)P(F) = \frac{11}{25}$

$\Rightarrow P(E) + P(F) - 2P(E).P(F) = \frac{11}{25}$ ...(2)

and $P(\bar{E} \cap \bar{F}) = \frac{2}{25} \Rightarrow P(\bar{E})\ P(\bar{F}) = \frac{2}{25}$

$\Rightarrow [1 - P(E)][1 - P(F)] = \frac{2}{25}$

$\Rightarrow 1 - P(E) - P(F) + P(E)P(F) = \frac{2}{25}$ ...(3)

Adding equation (2) and (3) we get

$1 - P(E)P(F) = \frac{13}{25}$ or $P(E)P(F) = \frac{12}{25}$ ...(4)

Using the result in equation (2) we get

$P(E) + P(F) = \frac{35}{25}$ ...(5)

Solving (4) and (5) we get

$P(E) = \frac{3}{5}$ and $P(F) = \frac{4}{5}$ or $P(E) = \frac{4}{5}$ and $P(F) = \frac{3}{5}$

$\therefore$ (a) and (d) are the correct options.

17. **(b,d)**

We have $P(X_1) = \frac{1}{2}$, $P(X_2) = \frac{1}{4}$, $P(X_3) = \frac{1}{4}$

$P(X) = P$(at least 2 engines are functioning)

$= P(X_1 \cap X_2 \cap X_3^C) + P(X_1 \cap X_2^C \cap X_3)$

$+ P(X_1^C \cap X_2 \cap X_3) + P(X_1 \cap X_2 \cap X_3)$

$= \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} + \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{4}$

(a) $P(X_1^C / X) = \frac{P(X_1^C \cap X)}{P(X)} = \frac{P(X_1^C \cap X_2 \cap X_3)}{P(X)}$

$= \frac{\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}}{\frac{1}{4}} = \frac{1}{8}$

(b) P [Exactly two engines are functioning $/X$]

$= \frac{P[(\text{Exactly two engines are functioning}) \cap X]}{P(X)}$

$= \frac{P(X_1^C \cap X_2 \cap X_3) + P(X_1 \cap X_2^C \cap X_3) + P(X_1 \cap X_2 \cap X_3^C)}{P(X)}$

$= \frac{\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}}{\frac{1}{4}} = \frac{7}{8}$

(c) $P(X/X_2) = \frac{P(X \cap X_2)}{P(X_2)}$

$= \frac{P(X_1 \cap X_2 \cap X_3) + P(X_1^C \cap X_2 \cap X_3) + P(X_1 \cap X_2 \cap X_3^C)}{P(X_2)}$

$= \frac{\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}}{\frac{1}{4}} = \frac{5}{8}$

(d) $P(X/X_1) = \frac{P(X \cap X_1)}{P(X_1)}$

$= \frac{P(X_1 \cap X_2 \cap X_3) + P(X_1 \cap X_2^C \cap X_3) + P(X_1 \cap X_2 \cap X_3^C)}{P(X_1)}$

$= \frac{\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}}{\frac{1}{2}} = \frac{7}{16}$

18. **(a, b)**

We know $P(X/Y) = \frac{P(X \cap Y)}{P(Y)} \Rightarrow \frac{1}{2} = \frac{1/6}{P(Y)} \Rightarrow P(Y) = \frac{1}{3}$

Similarly, $P(Y/X) = \frac{P(X \cap Y)}{P(X)} \Rightarrow \frac{1}{3} = \frac{1/6}{P(X)} \Rightarrow P(X) = \frac{1}{2}$

$\therefore \quad P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$

Also $P(X \cap Y) = P(X)P(Y)$

$\Rightarrow$ X and Y are independent events.

$\therefore$ X^C and Y are also independent events.

$\therefore$ $P(X^C \cap Y) = P(X^C) \times P(Y) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

19. **(a)** P (atleast one of them solves the problem)

$= 1 - P$ (none of them solves it)

$= 1 - \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} \times \frac{7}{8} = 1 - \frac{21}{256} = \frac{235}{256}$

E. Subjective Problems

1. To draw 2 black, 4 white and 3 red balls in order is same as arranging two black balls at first 2 places, 4 white at next 4 places, (3rd to 6th place) and 3 red at still next 3 places (7th to 9th place), i.e., $B_1B_2\ W_1W_2W_3W_4R_1\ R_2\ R_3$, which can be done in $2! \times 4! \times 3!$ ways. And total ways of arranging all $2 + 4 + 3 = 9$ balls is 9!

$\therefore$ Required probability $= \frac{2! \times 4! \times 3!}{9!} = \frac{1}{1260}$

2. (i) 6 boys and 6 girls sit in a row randomly.

Total ways of their seating = 12!

No. of ways in which all the 6 girls sit together $= 6! \times 7!$

(considering all 6 girls as one person)

$\therefore$ Probability of all girls sitting together

$$=\frac{6!\times 7!}{12!}=\frac{720}{12\times 11\times 10\times 9\times 8}=\frac{1}{132}$$

(ii) Staring with boy, boys can sit in 6! ways leaving one place between every two boys and one a last.

B _ B _ B _ B _ B _ B _

These left over places can be occupied by girls in 6! ways.

$\therefore$ If we start with boys. no. of ways of seating boys and girls alternately $= 6!\times 6!$

In the similar manner, if we start with girl, no. of ways of seating boys and girls alternately

$= 6!\times 6!$

G _ G _ G _ G _ G _ G _

Thus total ways of alternate seating arrangements

$= 6!\times 6! + 6!\times 6!$

$= 2\times 6!\times 6!$

$\therefore$ Probability of making alternate seating arrangement for 6 boys and 6 girls

$$=\frac{2\times 6!\times 6!}{12!}=\frac{2\times 720}{12\times 11\times 10\times 9\times 8\times 7}=\frac{1}{462}$$

3. (a) Let us define the events as :

$E_1 \equiv$ First shot hits the target plane,

$E_2 \equiv$ Second shot hits the target plane

$E_3 \equiv$ third shot hits the target plane,

$E_4 \equiv$ fourth shot hits the target plane

then ATQ, $P(E_1)=0.4;\ P(E_2)=0.3;$

$P(E_3)=0.2;\ P(E_4)=0.1$

$\Rightarrow P(\bar{E}_1)=1-0.4=0.6;\ P(\bar{E}_2)=1-0.3=0.7$

$P(\bar{E}_3)=1-0.2=0.8;\ P(\bar{E}_4)=1-0.1=0.9$

(where $\bar{E}_1$ denotes not happening of E_1)

Now the gun hits the plane if at least one of the four shots hit the plane.

Also, P (at least one shot hits the plane).

$= 1 - P$ (none of the shots hits the plane)

$= 1-P(\bar{E}_1\cap\bar{E}_2\cap\bar{E}_3\cap\bar{E}_4)$

$= 1-P(\bar{E}_1).P(\bar{E}_2).P(\bar{E}_3).P(\bar{E}_4)$

[Using multiplication thm for independent events]

$= 1-0.6\times 0.7\times 0.8\times 0.9 = 1-0.3024 = 0.6976$

4. Let A denote the event that the candidate A is selected and B the event that B is selected. It is given that

$P(A)=0.5$ ...(1)

$P(A\cap B)\le 0.3$ ...(2)

Now, $P(A)+P(B)-P(A\cap B)=P(A\cup B)\le 1$

or $0.5+P(B)-P(A\cap B)\le 1$ [Using (1)]

or $P(B)\le 0.5+P(A\cap B)\le 0.5+0.3$ [Using (2)]

or $P(B)\le 0.8\ \therefore P(B)$ can not be 0.9

5. We must have one ace in $(n-1)$ attempts and one ace in the nth attempt. The probability of drawing one ace in first

$(n-1)$ attempts is $\dfrac{{}^4C_1\times{}^{48}C_{n-2}}{{}^{52}C_{n-1}}$ and other one ace in the

nth attempt is, $\dfrac{{}^3C_1}{[52-(n-1)]}=\dfrac{3}{53-n}$

Hence the required probability,

$$=\frac{4.48!}{(n-2)!(50-n)!}\times\frac{(n-1)!(53-n)}{52!}\times\frac{3}{53-n}$$

$$=\frac{(n-1)(52-n)(51-n)}{50.49.17.13}$$

6. Given that

$P(A)=0.3, P(B)=0.4, P(C)=0.8$

$P(AB)=0.08, P(AC)=0.28, P(ABC)=0.09$

$P(A\cup B\cup C)\ge 0.75$

To find $P(BC)=x$ (say)

Now we know,

$P(A\cup B\cup C)=P(A)+P(B)+P(C)$

$-P(AB)-P(BC)-P(CA)+P(ABC)$

$\Rightarrow P(A\cup B\cup C)=0.3+0.4+0.8$

$-0.08-x-0.28+0.09=1.23-x$

Also we have,

$P(A\cup B\cup C)\ge 0.75$ and $P(A\cup B\cup C)\le 1$

$\therefore\ 0.75\le P(A\cup B\cup C)\le 1$

$\Rightarrow\ 0.75\le 1.23-x\le 1$

$\Rightarrow\ 0.23\le x\le 0.48$

7. Let $P(A)$ denotes the prob. of people reading newspaper A and $P(B)$ that of people reading newspaper B

Then, $P(A)=\dfrac{25}{100}=0.25$

$P(B)=\dfrac{20}{100}=0.20,\quad P(AB)=\dfrac{8}{100}=0.08$

Prob. of people reading the newspaper A but not $B=P(AB^c)$

$=P(A)-P(AB)=0.25-0.08=0.17$

Similarly, $P(A^cB)=P(B)-P(AB)=0.20-0.08=0.12$

Let E be the event that a person reads an advertisement.

Therefore, ATQ, $P(E/AB^c)=\dfrac{30}{100}$; $P(E/A^cB)=\dfrac{40}{100}$

$P(E/AB)=\dfrac{50}{100}$

$\therefore$ By total prob. theorem (as AB^c, A^cB and AB are mutually exclusive)

$P(E)=P(E/AB^c)P(AB^c)+P(E/A^cB)\ P(A^cB)$

$+P(E/AB).P(AB)$

$$=\frac{30}{100}\times 0.17+\frac{40}{100}\times 0.12+\frac{50}{100}\times 0.08$$

$=0.051+0.048+0.04=0.139.$

Thus the population that reads an advertisement is 13.9%.

8. The total number of ways of ticking the answers in any one attempt $=2^4-1=15.$

The student is taking chance at ticking the correct answer, It is reasonable to assume that in order to derive maximum benefit, the three solutions which he submit must be all different.

$\therefore$ $n=$ total no. of ways $={}^{15}C_3$

$m=$ the no. of ways in which the correct solution is excluded $={}^{14}C_3$

Hence the required probability $=1-\dfrac{{}^{14}C_3}{{}^{15}C_3}=1-\dfrac{4}{5}=\dfrac{1}{5}$

9. Let A_1 be the event that the lot contains 2 defective articles and A_2 the event that the lot contains 3 defective articles. Also let A be the event that the testing procedure ends at the twelth testing. Then according to the question :

$P(A_1)=0.4$ and $P(A_2)=0.6$
Since $0<P(A_1)<1, 0<P(A_2)<1$, and $P(A_1)+P(A_2)=1$
$\therefore$ The events A_1, A_2 form a partition of the sample space. Hence by the theorem of total probability for compound events, we have
$P(A)=P(A_1)P(A/A_1)+P(A_2)P(A/A_2)$ ...(1)
Here $P(A/A_1)$ is the probability of the event the testing procedure ends at the twelth testing when the lot contains 2 defective articles. This is possible when out of 20 articles, first 11 draws must contain 10 non defective and 1defective article and 12th draw must give a defective article.

$$\therefore \quad P(A/A_1)=\frac{{}^{18}C_{10}\times{}^{2}C_1}{{}^{20}C_{11}}\times\frac{1}{9}=\frac{11}{190}$$

Similarly, $P(A/A_2)=\dfrac{{}^{17}C_9\times{}^{3}C_1}{{}^{20}C_{11}}\times\dfrac{1}{9}=\dfrac{11}{228}$

Now substituting the values of $P(A/A_1)$ and $P(A/A_2)$ in eq. (1), we get

$$P(A)=0.4\times\frac{11}{190}+0.6\times\frac{11}{228}=\frac{11}{475}+\frac{11}{380}=\frac{99}{1900}$$

10. Since the man is one step away from starting point means that either
(i) man has taken 6 steps forward and 5 steps backward.
or (ii) man has taken 5 steps forward and 6 steps backward.
Taking movement 1 step forward as success and 1 step backward as failure.
$\therefore$ p = Probability of success = 0.4
and q = Probability of failure = 0.6
$\therefore$ Required probability $=P(X=6 \text{ or } X=5)$
$=P(X=6)+P(X=5)$
$={}^{11}C_6p^6q^5+{}^{11}C_5p^5q^6$
$={}^{11}C_5(p^6q^5+p^5q^6)={}^{11}C_5(p+q)(p^5q^5)$
$=\dfrac{11.10.9.8.7}{1.2.3.4.5}(0.4+0.6)(0.4\times0.6)^5$
$=462\times1\times(0.24)^5=0.37$
Hence the required prob. = 0.37

11. Here the total number of coins is $N+7$. Therefore the total number of ways of choosing 5 coins out of $N+7$ is ${}^{N+7}C_5$.
Let E denotes the event that the sum of the values of the coins is less than one rupee and fifty paise.
Then E' denotes the event that the total value of the five coins is equal to or more than one rupee and fifty paise.
The number of cases favourable to E' is
$={}^{2}C_1\times{}^{5}C_4\times{}^{N}C_0+{}^{2}C_2\times{}^{5}C_3\times{}^{N}C_0+{}^{2}C_2\times{}^{5}C_2\times{}^{N}C_1$
$=2\times5+10+10N=10(N+2)$

$$\therefore P(E')=\frac{10(N+2)}{{}^{N+7}C_5}\Rightarrow P(E)=1-P(E')=1-\frac{10(N+2)}{{}^{N+7}C_5}$$

12. The probability p_1 (say) of winning the best of three games is = the prob. of winning two games + the prob. of winning three games.
$={}^{3}C_2(0.6)(0.4)^2+{}^{3}C_3(0.4)^3$ [Using Binomial distribution]
Similarly the probability of winning the best five games is p_2 (say) = the prob. of winning three games + the prob. of winning four games + the prob. of winning 5 games

$$={}^{5}C_3(0.6)^2(0.4)^3+{}^{5}C_4(0.6)(0.4)^4+{}^{5}C_5(0.4)^5$$

We have $p_1=0.288+0.064=0.352$
and $p_2=0.2304+0.0768+0.01024=0.31744$
As $p_1>p_2$
$\therefore$ A must choose the first offer i.e. best of three games.

13. Let $A=\{a_1,a_2,a_3,\ldots,a_n\}$
For each a_i, $1\le i\le n$, there aries 4 cases
(i) $a_i\in P$ and $a_i\in Q$ (ii) $a_i\notin P$ and $a_i\in Q$
(iii) $a_i\in P$ and $a_i\notin Q$ (iv) $a_i\notin P$ and $a_i\notin Q$
$\therefore$ Total no. of ways of choosing P and Q is 4^n. Here case (i) is not favourable as $P\cap Q=\phi$
$\therefore$ For each element there are 3 favourable cases and hence total no. of favourable cases = 3^n.

Hence prob. $(P\cap Q=\phi)=\dfrac{3^n}{4^n}=\left(\dfrac{3}{4}\right)^n$.

14. Let us define the events :
$A_1\equiv$ the examinee guesses the answer,
$A_2\equiv$ the examinee copies the answer
$A_3\equiv$ the examinee knows the answer,
$A\equiv$ the examinee answers correctly.

Then, $P(A_1)=\dfrac{1}{3}$; $P(A_2)=\dfrac{1}{6}$

As any one happens out of A_1, A_2, A_3, these are mutually exclusive and exhaustive events.
$\therefore$ $P(A_1)+P(A_2)+P(A_3)=1$

$$\Rightarrow P(A_3)=1-\frac{1}{3}-\frac{1}{6}=\frac{6-2-1}{6}=\frac{3}{6}=\frac{1}{2}$$

Also we have, $P(A/A_1)=\dfrac{1}{4}$

[$\because$ out of 4 choices only one is correct.] $P(A/A_2)=\dfrac{1}{8}$
(given) $P(A/A_3)=1$
[If examinee knows the ans., it is correct. i.e. true event]
To find $P(A_3/A)$. By Baye's thm, $P(A_3/A)$

$$=\frac{P(A/A_3)P(A_3)}{P(A/A_1)P(A_1)+P(A/A_2)P(A_2)+P(A/A_3)P(A_3)}$$

$$=\frac{1.\frac{1}{2}}{\frac{1}{4}.\frac{1}{3}+\frac{1}{8}.\frac{1}{6}+1.\frac{1}{2}}=\frac{1/2}{\frac{29}{48}}=\frac{1}{2}\times\frac{48}{29}=\frac{24}{29}$$

15. Let X = defective and Y = non defective. Then all possible outcomes are $\{XX, XY, YX, YY\}$

Also $P(XX)=\dfrac{50}{100}\times\dfrac{50}{100}=\dfrac{1}{4}$,

$P(XY)=\dfrac{50}{100}\times\dfrac{50}{100}=\dfrac{1}{4}$ $P(YX)=\dfrac{50}{100}\times\dfrac{50}{100}=\dfrac{1}{4}$,

$P(YY)=\dfrac{50}{100}\times\dfrac{50}{100}=\dfrac{1}{4}$

Here, $A=XX\cup XY$; $B=XY\cup YY$; $C=XX\cup YY$

$\therefore$ $P(A)=P(XX)+P(XY)=\dfrac{1}{4}+\dfrac{1}{4}=\dfrac{1}{2}$

$\therefore$ $P(B)=P(XY)+P(YX)=\dfrac{1}{4}+\dfrac{1}{4}=\dfrac{1}{2}$

$P(C)=P(XX)+P(YY)=\dfrac{1}{4}+\dfrac{1}{4}=\dfrac{1}{2}$

Now, $P(AB) = P(XY) = \frac{1}{4} = P(A).P(B)$

$\therefore$ A and B are independent events.

$P(BC) = P(YX) = \frac{1}{4} = P(B).P(C)$

$\therefore$ B and C are independent events.

$P(CA) = P(XX) = \frac{1}{4} = P(C).P(A)$

$\therefore$ C and A are independent events.

$P(ABC) = 0$ (impossible event)

$\neq P(A)P(B)P(C)$

$\therefore$ A, B, C are dependent events.

Thus we can conclude that A, B, C are pairwise independent but A, B, C are dependent events.

16. The given numbers are 00, 01, 02, ..., 99. These are total 100 numbers, out of which the numbers, the product of whose digits is 18, are 29, 36, 63 and 92.

$\therefore$ $p = P(E) = \frac{4}{100} = \frac{1}{25} \Rightarrow q = 1 - p = \frac{24}{25}$

From Binomial distribution

P (E occuring at least 3 times) = P (E occuring 3 times) + P (E occuring 4 times)

$${}^4C_3 p^3 q + {}^4C_4 p^4 = 4 \times \left(\frac{1}{25}\right)^3 \left(\frac{24}{25}\right) + \left(\frac{1}{25}\right)^4 = \frac{97}{(25)^4}$$

17. $E_1 \equiv$ number noted is 7, $E_2 \equiv$ number notes is 8, $H \equiv$ getting head on coin, $T \equiv$ getting tail on coin.

Then by total probability theorem,

$P(E_1) = P(H)P(E_1/H) + P(T)P(E_1/T)$

and $P(E_2) = P(H)P(E_2/H) + P(T)P(E_2/T)$

where $P(H) = \frac{1}{2}; P(T) = \frac{1}{2}$

$P(E_1/H)$ = prob. of getting a sum of 7 on two dice. Here favourable cases are {(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)}

$\therefore$ $P(E_1/H) = \frac{6}{36} = \frac{1}{6}$

Also $P(E_1/T)$ = prob. of getting '7' numbered card out of 11 cards $= \frac{1}{11}$.

$P(E_2/H)$ = Prob. of getting a sum of 8 on two dice. Here favourable cases are {(2, 6) (6, 2) (4, 4), (5, 3), (3, 5)}

$\therefore$ $P(E_2/H) = \frac{5}{36}$

$P(E_2/T)$ = prob. of getting '8' numbered card out of 11 cards $= \frac{1}{11}$

$$\therefore P(E_1) = \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{11} = \frac{1}{12} + \frac{1}{22} = \frac{11+6}{132} = \frac{17}{132}$$

$$P(E_2) = \frac{1}{2} \times \frac{5}{36} + \frac{1}{2} \times \frac{1}{11} = \frac{1}{2}\left[\frac{55+36}{396}\right] = \frac{91}{792}$$

Now E_1 and E_2 are mutually exclusive events therefore

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) = \frac{17}{132} + \frac{91}{792}$$

$$= \frac{102+91}{792} = \frac{193}{792} = 0.2436.$$

18. We have 14 seats in two vans. And there are 9 boys and 3 girls. The no. of ways of arranging 12 people on 14 seats without restriction is ${}^{14}P_{12} = \frac{14!}{2!} = 7(13!)$

Now the no. of ways of choosing back seats is 2. And the no. of ways of arranging 3 girls on adjacent seats is 2 (3!). And the no. of ways of arranging 9 boys on the remaining 11 seats is ${}^{11}P_9$

Therefore, the required number of ways

$$= 2.(2.3!).\ {}^{11}P_9 = \frac{4.3!.11!}{2!} = 12!$$

Hence, the probability of the required event $= \frac{12!}{7.13!} = \frac{1}{91}$

19. The required probability = 1 – (probability of the event that the roots of $x^2 + px + q = 0$ are non-real if and only if $p^2 - 4q < 0$ i.e. if $p^2 < 4q$.

We enumerate the possible values of p and q, for which this can happen in the following table.

q	p	Number of pairs of p,q
1	1	1
2	1, 2	2
3	1, 2, 3	3
4	1, 2, 3	3
5	1, 2, 3, 4	4
6	1, 2, 3, 4	4
7	1, 2, 3, 4, 5	5
8	1, 2, 3, 4, 5	5
9	1, 2, 3, 4, 5	5
10	1, 2, 3, 4, 5, 6	6

Thus, the number of possible pairs = 38. Also, the total number of possible pairs is $10 \times 10 = 100$.

$\therefore$ The required probability $= 1 - \frac{38}{100} = 1 - 0.38 = 0.62$

20. Given that p is the prob. that coin shows a head then $1-p$ will be the prob. that coin shows a tail.

Now, $\alpha = P$ (A gets the 1st head in 1st try) + P (A gets the 1st head in 2nd try) + ...

$\Rightarrow$ $\alpha = P(H) + P(T)P(T)P(T)P(H) + P(T)P(T)P(T)P(T)P(T)P(T)P(H)$

$= p + (1-p)^3 p + (1-p)^6 p + \ldots\ldots$

$$= p[1 + (1-p)^3 (1-p)^6 + \ldots\ldots] = \frac{p}{1-(1-p)^3}$$

Similarly $\beta = P$ (B gets the 1st head in 1st try) + P (B gets the 1st head in 2nd try) + ...

$= P(T)P(H) + P(T)P(T)P(T)P(T)P(H) + \ldots\ldots$

$$= (1-p)p + (1-p)^4 p + \ldots\ldots = \frac{(1-p)p}{1-(1-p)^3} \quad \ldots(ii)$$

From (i) and (ii) we get $\beta = (1-p)\alpha$

Also (i) and (ii) give expresion for α and β in terms of p.

Also $\alpha + \beta + \gamma = 1$ (exhaustive events and mutually exclusive events)

$\Rightarrow$ $\gamma = 1 - \alpha - \beta = 1 - \alpha - (1-p)\alpha$

$= 1-(2-p)\,\alpha = 1-(2-p)\dfrac{p}{1-(1-p)^3}$

$= \dfrac{1-(1-p)^3-(2p-p^2)}{1-(1-p)^3}$

$= \dfrac{1-1+p^3+3p(1-p)-2p+p^2}{1-(1-p)^3}$

$= \dfrac{p^3-2p^2+p}{1-(1-p)^3} = \dfrac{p(p^2-2p+1)}{1-(1-p)^3} = \dfrac{p(1-p)^2}{1-(1-p)^3}$

21. The number of ways in which $P_1, P_2, \ldots, P_8$ can be paired in four pairs $= \dfrac{1}{4!}\times {}^8C_2\times {}^6C_2\times {}^4C_2\times {}^2C_2 = 105$

Now, at least two players certainly reach the second round in between P_1, P_2 and P_3. And P_4 can reach in final if exactly two players play against each other in between P_1, P_2, P_3 and remaining player will play against one of the players from P_5, P_6, P_7, P_8 and P_4 plays against one of the remaining three from P_5, P_6, P_7, P_8.

This can be possible in ${}^3C_2\times {}^4C_1\times {}^3C_1 = 36$ ways

$\therefore$ Prob. that P_4 and exactly one of $P_5 \ldots P_8$ reach second round $= \dfrac{36}{105} = \dfrac{12}{35}$

If P_1, P_i, P_4 and P_j where $i = 2$ or 3 and $j = 5$ or 6 or 7 reach the second round then they can be paired in 2 pairs in $\dfrac{1}{2!}\times {}^4C_2\times {}^2C_2 = 3$ ways

But P_4 will reach the final if P_1 plays against P_i and P_4 plays against P_j.

Hence the prob. that P_4 reach the final round from the second $= \dfrac{1}{3}$.

$\therefore$ prob. that P_4 reach the final is $\dfrac{12}{35}\times\dfrac{1}{3} = \dfrac{4}{35}$

22. Given that the probability of showing head by a coin when tossed $= p$

$\therefore$ Prob. of coin showing a tail $= 1-p$

Now p_n = prob. that no two or more consecutive heads occur when tossed n times.

$\therefore$ p_1 = prob. of getting one or more or no head = prob. of H or $T = 1$

Also p_2 = prob. of getting one H or no H

$= P(HT) + P(TH) + P(TT)$

$= p(1-p) + p(1-p)p + (1-p)(1-p) = 1-p^2$, For $n \ge 3$

p_n = prob. that no two or more consecutive heads occur when tossed n times.

= P (last out come is T) P (no two or more consecutive heads in $(n-1)$ throw) + P (last out come is H) P ($(n-1)$th throw results in a T) P (no two or more consecutive heads in $(n-2)$ n throws) $= (1-p)P_{n-1} + p(1-p)p_{n-2}$

Hence Proved.

23. Let W_1 (B_1) be the event that a white (a back) ball is drawn in the first draw and let W be the event that a white ball is drawn in the second draw. Then

$P(W) = P(B_1)\,.\,P(W/B_1) + P(W_1).\ P(W/W_1)$

$= \dfrac{n}{m+n}\cdot\dfrac{m}{m+n+k} + \dfrac{m}{m+n}\cdot\dfrac{m+k}{m+n+k}$

$= \dfrac{m(n+m+k)}{(m+n)(m+n+k)} = \dfrac{m}{m+n}$

24. The total no. of outcomes $= 6^n$

We can choose three numbers out of 6 in ${}^6C_3$ ways. By using three numbers out of 6 we can get 3^n sequences of length n. But these include sequences of length n which use exactly two numbers and exactly one number.

The number of n-sequences which use exactly two numbers $= {}^3C_2[2^n - 1^n - 1^n] = 3(2^n - 2)$ and the number of n sequences which are exactly one number $= ({}^3C_1)(1^n) = 3$.

Thus , the number of sequences, which use exactly three numbers

$= {}^6C_3[3^n - 3(2^n - 2) - 3] = {}^6C_3[3^n - 3(2^n) + 3]$

$\therefore$ Probability of the required event, $= \dfrac{{}^6C_3[3^n - 3(2^n)+3]}{6^n}$

25. Let E_1 be the event that the coin drawn is fair and E_2 be the event that the coin drawn is biased.

$\therefore\quad P(E_1) = \dfrac{m}{N}$ and $P(E_2) = \dfrac{N-m}{N}$

A is the event that on tossing the coin the head appears first and then appears tail.

$\therefore\quad P(A) = P(E_1\cap A) + P(E_2\cap A)$

$= P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$

$= \dfrac{m}{N}\left(\dfrac{1}{2}\right)^2 + \left(\dfrac{N-m}{N}\right)\left(\dfrac{2}{3}\right)\left(\dfrac{1}{3}\right) \quad \ldots(1)$

We have to find the probability that A has happened because of E_1

$\therefore\quad P(E_1/A) = \dfrac{P(E_1\cap A)}{P(A)}$

$= \dfrac{\dfrac{m}{N}\left(\dfrac{1}{2}\right)^2}{\dfrac{m}{N}\left(\dfrac{1}{2}\right)^2 + \dfrac{N-m}{N}\left(\dfrac{2}{3}\right)\left(\dfrac{1}{3}\right)}$ (by (1))

$= \dfrac{m/4}{m/4 + \dfrac{2(N-m)}{9}} = \dfrac{9m}{m+8N}$

26. Let us consider

$E_1 \equiv$ event of passing I exam.

$E_2 \equiv$ event of passing II exam.

$E_3 \equiv$ event of passing III exam.

Then a student can qualify in anyone of following ways

1. He passes first and second exam.
2. He passes first, fails in second but passes third exam.
3. He fails in first, passes second and third exam.

$\therefore$ Required probability

$= P(E_1)P(E_2/E_1) + P(E_1)P(\bar{E}_2/E_1)P(E_3/\bar{E}_2) + P(\bar{E}_1)P(E_2/\bar{E}_1)P(E_3/E_2)$

[as an event is dependent on previous one]

$= p.p + p.(1-p).\dfrac{p}{2} + (1-p).\dfrac{p}{2}.p$

$= p^2 + \dfrac{p^2}{2} - \dfrac{p^3}{2} + \dfrac{p^2}{2} - \dfrac{p^3}{2} = 2p^2 - p^3$

27. Let us consider the events

$E_1 \equiv A$ hits B Then $P(E_1) = 2/3$

$E_2 \equiv B$ hits A $P(E_2) = 1/2$

$E_3 \equiv C$ hits A $P(E_3) = 1/3$

$E \equiv A$ is hit

$$P(E) = P(E_2 \cup E_3) = 1 - P(\bar{E}_2 \cap \bar{E}_3)$$

$$= 1 - P(\bar{E}_2).P(\bar{E}_3) = 1 - \frac{1}{2}.\frac{2}{3} = \frac{2}{3}$$

To find $P(E_2 \cap \bar{E}_3 / E)$

$$= \frac{P(E_2 \cap \bar{E}_3)}{P(E)} \quad [\because P(E_2 \cap \bar{E}_3 \cap E) = P(E_2 \cap \bar{E}_3) \text{ i.e.,}$$

B hits A and A is hit $= B$ hits A]

$$= \frac{P(E_2).P(\bar{E}_3)}{P(E)} = \frac{1/2 \times 2/3}{2/3} = \frac{1}{2}$$

28. Given that A and B are two independent events. C is the event in which exactly of A or B occurs.

Let $P(A) = x$, $P(B) = y$

then $P(C) = P(A \cap \bar{B}) + P(\bar{A} \cap B) = P(A)P(\bar{B}) + P(\bar{A})P(B)$

[$\because$ If A and B are independent so are 'A and $\bar{B}$' and '$\bar{A}$ and B'.]

$\Rightarrow P(C) = x(1-y) + y(1-x)$ ...(1)

Now consider, $P(A \cup B)\, P(\bar{A} \cap \bar{B})$

$= [P(A) + P(B) - P(A)P(B)]\,[P(\bar{A})P(\bar{B})]$

$= (x + y - xy)(1-x)(1-y)$

$= (x+y)(1-x)(1-y) - xy(1-x)(1-y) \le (x+y)(1-x)(1-y)$

[$\because x, y \in (0,1)$]

$= x(1-x)(1-y) + y(1-x)(1-y)$

$= x(1-y) + y(1-x) - x^2(1-y) - y^2(1-x) \le x(1-y) + y(1-x)3$

$= P(C)$ [Using eq^n (1)]

Thus $P(C) \ge P(A \cup B) P(\bar{A} \cap \bar{B})$ is proved.

29. Let us define the following events

$A \equiv$ 4 white balls are drawn in first six draws

$B \equiv$ 5 white balls are drawn in first six draws

$C \equiv$ 6 white balls are drawn in first six draws

$E \equiv$ exactly one white ball is drawn in next two draws (i.e. one white and one red)

Then $P(E) = P(E/A)\,P(A) + P(E/B)\,P(B) + P(E/C)\,P(C)$

But $P(E/C) = 0$ [As there are only 6 white balls in the bag.]

$$P(E) = P(E/A)\,P(A) + P(E/B)\,P(B)$$

$$= \frac{{}^{10}C_1 \times {}^{2}C_1}{{}^{12}C_2} \cdot \frac{{}^{12}C_2 \times {}^{6}C_4}{{}^{18}C_6} + \frac{{}^{11}C_1 \times {}^{1}C_1}{{}^{12}C_2} \cdot \frac{{}^{12}C_1 \times {}^{6}C_5}{{}^{18}C_6}$$

30. Let us define the following events

$C \equiv$ person goes by car,

$S \equiv$ person goes by scooter,

$B \equiv$ person goes by bus,

$T \equiv$ person goes by train,

$L \equiv$ person reaches late

Then we are given in the question

$$P(C) = \frac{1}{7}; P(S) = \frac{3}{7}; P(B) = \frac{2}{7}; P(T) = \frac{1}{7}$$

$$P(L/C) = \frac{2}{9}; P(L/S) = \frac{1}{9}; P(L/B) = \frac{4}{9}; P(L/T) = \frac{1}{9}$$

To find the prob. $P(C/\bar{L})$ [$\because$ reaches in time $\equiv$ not late]

Using Baye's theorem

$$P(C/\bar{L}) = \frac{P(\bar{L}/C)P(C)}{P(\bar{L}/C)P(C) + P(\bar{L}/S)P(S) + P(\bar{L}/B)P(B) + A(\bar{L}/T)P(T)} \quad \text{...(i)}$$

Now, $P(\bar{L}/C) = 1 - \frac{2}{9} = \frac{7}{9}$; $P(\bar{L}/S) = 1 - \frac{1}{9} = \frac{8}{9}$

$P(\bar{L}/B) = 1 - \frac{4}{9} = \frac{5}{9}$; $P(\bar{L}/T) = 1 - \frac{1}{9} = \frac{8}{9}$

Substituting these values in eqn. (i) we get

$$P(C/\bar{L}) = \frac{\frac{7}{9} \times \frac{1}{7}}{\frac{7}{9} \times \frac{1}{7} + \frac{8}{9} \times \frac{3}{7} + \frac{5}{9} \times \frac{2}{7} + \frac{8}{9} \times \frac{1}{7}}$$

$$= \frac{7}{7 + 24 + 10 + 8} = \frac{7}{49} = \frac{1}{7}$$

G. Comprehension Based Questions

1. **(b)** $P(u_i) \propto i \Rightarrow P(u_i) = ki$, But $\sum P(u_i) = 1$

$$\Rightarrow \sum ki = 1 \Rightarrow k \sum i = 1 \Rightarrow k = \frac{2}{n(n+1)} \Rightarrow P(u_i) = \frac{2i}{n(n+1)}$$

By total prob. theorem

$$P(w) = \sum_{i=1}^{n} P(u_i) P(w/u_i) = \sum_{i=1}^{n} \frac{2i}{n(n+1)} \times \frac{i}{n+1}$$

$$= \frac{2}{n(n+1)^2} \cdot \frac{n(n+1)(2n+1)}{6} = \frac{2n+1}{3n+3}$$

$$\therefore \lim_{n \to \infty} P(w) = \lim_{n \to \infty} \frac{2n+1}{3n+3} = \lim_{n \to \infty} \frac{2 + 1/n}{3 + 3/n} = \frac{2}{3}$$

2. **(a)** $P(u_i) = c$

Using Baye's theorem, $P(u_n/w) = \dfrac{P(w/u_n)P(u_n)}{\sum_{i=1}^{n} P(w/u_i)P(u_i)}$

$$= \frac{c \times \frac{n}{n+1}}{c\left[\frac{1}{n+1} + \frac{2}{n+1} + \ldots + \frac{n}{n+1}\right]} = \frac{n}{n+1} \times \frac{n+1}{\frac{n(n+1)}{2}} = \frac{2}{n+1}$$

3. **(b)** $P(w/E) = \dfrac{P(w \cap E)}{P(E)}$

$$= \frac{\frac{1}{n} \times \frac{2}{n+1} + \frac{1}{n} \times \frac{4}{n+1} + \frac{1}{n} \times \frac{6}{n+1} + \ldots + \frac{1}{n} \times \frac{n}{n+1}}{\frac{1}{n} + \frac{1}{n} + \ldots + \frac{1}{n}\left(\frac{n}{2} \text{times}\right)}$$

$$= \frac{\frac{2}{n(n+1)}\left[1 + 2 + 3 \ldots + \frac{n}{2}\right]}{\frac{1}{n} \times \frac{n}{2}} \quad (n \text{ being even})$$

$$=\frac{4}{n(n+1)}\left[\frac{\frac{n}{2}\left(\frac{n}{2}+1\right)}{2}\right]=\frac{n+2}{2(n+1)}$$

4. **(a)** $P(X=3)$ = (probability of not a six in first chance) × (probability of not a six in second chance) × (probability of a six in third chance)

$$=\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}=\frac{25}{216}$$

5. **(b)** $P(X\geq 3)=1-(X<3)\ =1-[P(X=1)+P(X=2)]$

$$=1-\left[\frac{1}{6}+\frac{5}{6}\times\frac{1}{6}\right]\ =1-\frac{11}{36}=\frac{25}{36}$$

6. **(d)** Let us define the events

$A\equiv X\geq 6$ and $B\equiv X>3$ so that $A\cap B\equiv X\geq 6\equiv A$

Now $P(A)=\left(\frac{5}{6}\right)^5\times\frac{1}{6}+\left(\frac{5}{6}\right)^6\times\frac{1}{6}+....\infty$

$$=\left(\frac{5}{6}\right)^5\times\frac{1}{6}\left[1+\frac{5}{6}+\left(\frac{5}{6}\right)^2+....\infty\right]=\left(\frac{5}{6}\right)^5\times\frac{1}{6}\times\frac{1}{1-\frac{5}{6}}=\left(\frac{5}{6}\right)^5$$

and $P(B)=\left(\frac{5}{6}\right)^3\times\frac{1}{6}+\left(\frac{5}{6}\right)^4\times\frac{1}{6}+....\infty=\left(\frac{5}{6}\right)^3$

$$\therefore\quad P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)}{P(B)}=\frac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3}=\frac{25}{36}$$

7. **(b)** P (white) = P (H ∩ white) + P (T ∩ white)
= P (H) P (white/H) + P (T) P (white/T)

$$=\frac{1}{2}\left\{\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{2}\right\}+\frac{1}{2}$$

$$\times\left\{\frac{^3C_2}{^5C_2}\times 1+\frac{^2C_2}{^5C_2}\times\frac{1}{3}+\frac{^3C_1.^2C_1}{^5C_2}\times\frac{2}{3}\right\}$$

$$=\frac{1}{2}\times\frac{8}{10}+\frac{1}{2}\times\left(\frac{3}{10}+\frac{1}{30}+\frac{12}{30}\right)=\frac{4}{10}+\frac{11}{30}=\frac{23}{30}$$

8. **(d)** $$\text{P(H/white)}=\frac{P(H\cap\text{white})}{\text{P(white)}}=\frac{\frac{1}{2}\left[\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{2}\right]}{\frac{23}{30}}$$

$$=\frac{\frac{4}{10}}{\frac{23}{30}}=\frac{12}{23}$$

9. **(a)** Probability that all balls are of same colour
= P (all red) + P (all white) + P (all black)

$$=\frac{3}{6}\times\frac{3}{9}\times\frac{4}{12}+\frac{1}{6}\times\frac{2}{9}\times\frac{3}{12}+\frac{2}{6}\times\frac{4}{9}\times\frac{5}{12}=\frac{82}{648}$$

10. **(d)** B_1 [1W, 3R, 2B] B_2 [2W, 3R, 4B] B_3 [3W, 4R, 5B]

Let E_1, E_2, E_3 be the events that bag B_1, B_2 and B_3 is selected respectively.
Let E be the event that one white and one red ball is selected.
Then by baye's theorem,

$$P(E_2\backslash E)=\frac{P(E\backslash E_2)P(E_2)}{P(E\backslash E_1)P(E_1)+P(E\backslash E_2)P(E_2)+P(E\backslash E_3)P(E_3)}$$

$$=\frac{\frac{2\times 3}{9C_2}}{\frac{1\times 3}{^6C_2}+\frac{2\times 3}{^9C_2}+\frac{3\times 4}{^{12}C_2}}=\frac{55}{181}$$

11. **(b)** $x_1+x_2+x_3$ will be odd
If two are even and one is odd or all three are odd.
∴ Required probability
$=P(EEO)+P(EOE)+P(OEE)+P(OQO)$

$$=\frac{1}{3}\times\frac{2}{5}\times\frac{4}{7}+\frac{1}{3}\times\frac{3}{5}\times\frac{3}{7}+\frac{2}{3}\times\frac{2}{5}\times\frac{3}{7}+\frac{2}{3}\times\frac{3}{5}\times\frac{4}{7}$$

$$=\frac{8+9+12+24}{105}=\frac{53}{105}$$

12. **(c)** If x_1, x_2, x_3 are in AP then $2x_2=x_1+x_3$
∵ LHS is even, x_1 & x_3 can be both even or both odd.
x_1 and x_3 both can be even in 1 × 3 = 3 ways
x_1 and x_3 both can be odd in 2 × 4 = 8 ways
∴ Total favourable ways = 3 + 8 = 11
Also one number from each box can be drawn in 3 × 5 × 7 ways
∴ Total ways = 105

Hence required probability $=\frac{11}{105}$

13. **(a,b)** Let $E_1\equiv$ box I is selected
$E_2\equiv$ box II is selected
$E\equiv$ ball drawn is red

$$P(E_2/E)=\frac{\frac{n_3}{n_3+n_4}\times\frac{1}{2}}{\frac{n_1}{n_1+n_2}\times\frac{1}{2}+\frac{n_3}{n_3+n_4}\times\frac{1}{2}}=\frac{1}{3}$$

or $$\frac{\frac{n_3}{n_3+n_4}}{\frac{n_1}{n_1+n_2}+\frac{n_3}{n_3+n_4}}=\frac{1}{3}$$

On checking the options we find (a) and (b) are the correct options.

14. **(c, d)** $E_1\equiv$ Red ball is selected from box I
$E_2\equiv$ Black ball is selected from box I
$E\equiv$ Second ball drawn from box I is red
∴ $P(E)=P(E_1)P(E/E_1)+P(E_2)P(E/E_2)$

$$=\frac{n_1}{n_1+n_2}\times\frac{n_1-1}{n_1+n_2-1}+\frac{n_2}{n_1+n_2}\times\frac{n_1}{n_1+n_2-1}$$

On checking the options, we find (c) and (d) have the correct values.

For (Q. 15 - 16)
(X, Y) = {(6, 0), (4, 1), (3, 3), (2, 2), (4, 4), (0, 6)}

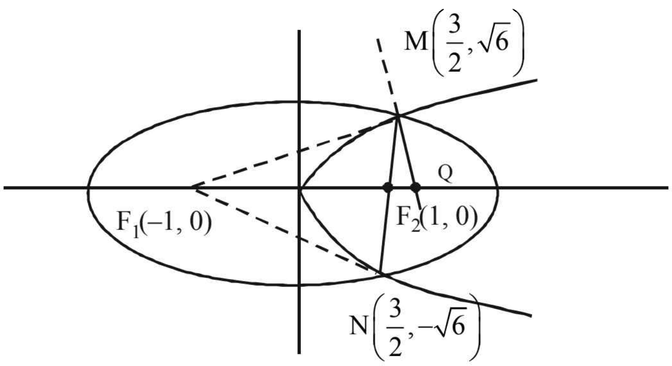
15. (b) P(X > Y) = P(T_1 wins 2 games or T_1 win one game other is a draw)

$$=\frac{1}{2}\times\frac{1}{2}+\left(\frac{1}{2}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{2}\right)$$

$$=\frac{1}{4}+\frac{1}{6}=\frac{5}{12}$$

16. (c) P(X = Y) = P(T_1 wins 1 game loses other game or both the games draw)

$$=\left(\frac{1}{2}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{2}\right)+\frac{1}{6}\times\frac{1}{6}=\frac{1}{3}+\frac{1}{36}=\frac{13}{36}$$



H. Assertion & Reason Type Questions

1. We know $P(H_i/E)=\frac{P(H_i\cap E)}{P(E)}=\frac{P(E/H_i)P(H_i)}{P(E)}$

$\Rightarrow P(H_i/E)\,P(E)=P(E/H_i)\,P(H_i)\Rightarrow P(E)=\frac{P(E/H_i)P(H_i)}{P(H_i/E)}$

Now given that $0 < P(E) < 1 \Rightarrow \quad 0<\frac{P(E/H_i)P(H_i)}{P(H_i/E)}<1$

$\Rightarrow$ $P(E/H_i)\,P(H_i) < P(H_i/E)$ But if $P(H_i\cap E)=0$ then $P(H_i/E) = P(E/H_i) = 0$

Then $P(E/H_i)\,P(H_i) < P(H_i/E)$ is not true.

$\therefore$ Statement -1 is not always true.

Also as $H_1, H_2, \ldots H_n$ are mutually exclusive and exhaustive events, therefore $\sum_{i=1}^{n}P(H_i)=1$. $\therefore$ Statement -2 is true.

2. The given system of equastions is
$ax + by = 0$ $cx + dy = 0$ where $a, b, c, d \in \{0, 1\}$
For the system to have unique solution,

$\frac{a}{c}\neq\frac{b}{d}$ i.e. $\begin{vmatrix}a & b\\ c & d\end{vmatrix}\neq 0$

This is so in each of the following cases -

$\begin{vmatrix}1 & 1\\ 1 & 0\end{vmatrix},\begin{vmatrix}1 & 1\\ 0 & 1\end{vmatrix},\begin{vmatrix}0 & 1\\ 1 & 1\end{vmatrix},\begin{vmatrix}1 & 0\\ 1 & 1\end{vmatrix},\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix},\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}$

$\therefore$ Favourable cases for the system to have unique solution = 6.

Also total possible cases for $\begin{vmatrix}a & b\\ c & d\end{vmatrix}=2^4 = 16$

($\because$ each entry can be either 0 or 1)

$\therefore$ Probability of the given system to have unique solution

$=\frac{6}{16}=\frac{3}{8}$ $\because$ Statment –1 is true.

$\because$ Homogeneous system of equations always has a solution (Trival solution $x = 0, y = 0$)

$\therefore$ The probability that the system of equations has a solution is 1.

Hence the statement-2 is true but is not a correct explanation of statement- 1.

I. Integer Value Correct Type

1. (6) Let $P(E_1) = x$; $P(E_2) = y$, $P(E_3) = z$
$P(\text{only } E_1) = x(1 - y)(1 - z) = \alpha$
$P(\text{only } E_2) = (1 - x)\,y\,(1 - z) = \beta$
$P(\text{only } E_3) = (1 - x)(1 - y)\,z = \gamma$
$P(\text{none}) = (1 - x)(1 - y)(1 - z) = p.$
Now given $(\alpha - 2\beta)\,p = \alpha\beta \Rightarrow x = 2y$
and $(\beta - 3r)p = 2\beta r$

$\Rightarrow y = 3z \;\therefore x = 6z$ Hence $\frac{P(E_1)}{P(E_3)}=\frac{x}{z}=6$

2. (8) $P(x \geq 2) \geq 0.96$
$\Rightarrow \quad 1 - P(x = 0) - P(x = 1) \geq 0.96$
$\Rightarrow \quad P(x = 0) + P(x = 1) \leq 0.04$

$\Rightarrow \quad \left(\frac{1}{2}\right)^n+n\left(\frac{1}{2}\right)^n\leq 0.04$

$\Rightarrow \quad \frac{n+1}{2^n}\leq\frac{1}{25}\Rightarrow\quad\frac{2^n}{n+1}\geq 25$

$\Rightarrow$ minimum value of n is 8.

Section-B JEE Main/ AIEEE

1. (a) $P(E_1)=\frac{1}{2}, P(E_2)=\frac{1}{3}$ and $P(E_3)=\frac{1}{4}$;

$P(E_1 U E_2 U E_3) = 1 - P(\bar{E}_1)P(\bar{E}_2)P(\bar{E}_3)$

$=1-\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)=1-\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=\frac{3}{4}$

2. (a) $P(A\cup B) = P(A) + P(B) - P(A\cap B)$;

$\Rightarrow \frac{3}{4}=1-P(\bar{A})+P(B)-\frac{1}{4}$

$\Rightarrow 1=1-\frac{2}{3}+P(B)\Rightarrow P(B)=\frac{2}{3}$;

Now, $P(\bar{A}\cap B)=P(B)-P(A\cap B)=\frac{2}{3}-\frac{1}{4}=\frac{5}{12}$.

3. **(d)** The event follows binomial distribution with $n=5, p=3/6=1/2$.
$q=1-p=1/2$.; $\therefore$ Variance $=npq=5/4$.

4. **(b)** $\left.\begin{array}{l} np=4 \\ npq=2 \end{array}\right\} \Rightarrow q=\frac{1}{2}, p=\frac{1}{2}, n=8$

$$p(X=1)={}^8C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^7=8.\frac{1}{2^8}=\frac{1}{2^5}=\frac{1}{32}$$

5. **(b)** $P(A)=\frac{3x+1}{3}, P(B)=\frac{1-x}{4}, P(C)=\frac{1-2x}{2}$

$\because$ For any event $E, 0 \le P(E) \le 1$

$$\Rightarrow 0 \le \frac{3x+1}{3} \le 1,\ 0 \le \frac{1-x}{4} \le 1 \text{ and } 0 \le \frac{1-2x}{2} \le 1$$

$$\Rightarrow -1 \le 3x \le 2, -3 \le x \le 1 \text{ and } -1 \le 2x \le 1$$

$$\Rightarrow -\frac{1}{3} \le x \le \frac{2}{3} \le -3 \le x \le 1, \text{ and } -\frac{1}{2} \le x \le \frac{1}{2}$$

Also for mutually exclusive events A, B, C,
$P(A \cup B \cup C)=P(A)+P(B)+P(C)$

$$\Rightarrow P(A \cup B \cup C)=\frac{3x+1}{3}+\frac{1-x}{4}+\frac{1-2x}{2}$$

$$\therefore 0 \le \frac{1+3x}{3}+\frac{1-x}{4}+\frac{1-2x}{2} \le 1$$

$$0 \le 13-3x \le 12 \Rightarrow 1 \le 3x \le 13 \Rightarrow \frac{1}{3} \le x \le \frac{13}{3}$$

Considering all inequations, we get

$$\max\left\{-\frac{1}{3},-3,-\frac{1}{2},\frac{1}{3}\right\} \le x \le \min\left\{\frac{2}{3},1,\frac{1}{2},\frac{13}{3}\right\}$$

$$\frac{1}{3} \le x \le \frac{1}{2} \Rightarrow x \in \left[\frac{1}{3},\frac{1}{2}\right]$$

6. **(a)** Let 5 horses are H_1, H_2, H_3, H_4 and H_5. Selected pair of horses will be one of the 10 pairs (i.e.; ${}^5C_2$): H_1H_2, H_1H_3, H_1H_4, H_1H_5, H_2H_3, H_2H_4, H_2H_5, H_3H_4, H_3H_5 and H_4H_5.
Any horse can win the race in 4 ways.
For example : Horses H_2 win the race in 4 ways H_1H_2, H_2H_3, H_2H_4 and H_2H_5.

Hence required probability $=\frac{4}{10}=\frac{2}{5}$

7. **(c)** A and B will contradict each other if one speaks truth and other false . So , the required

$$\text{Probability}=\frac{4}{5}\left(1-\frac{3}{4}\right)+\left(1-\frac{4}{5}\right)\frac{3}{4}$$

$$=\frac{4}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{3}{4}=\frac{7}{20}$$

8. **(b)** $P(E)=P(2 \text{ or } 3 \text{ or } 5 \text{ or } 7)$
$=0.23+0.12+0.20+0.07=0.62$
$P(F)=P(1 \text{ or } 2 \text{ or } 3)=0.15+0.23+0.12=0.50$
$P(E \cap F)=P(2 \text{ or } 3)=0.23+0.12=0.35$
$\therefore P(EUF)=P(E)+P(F)-P(E \cap F)$
$=0.62+0.50-0.35=0.77$

9. **(a)** mean $=np=4$ and variance $=npq=2$

$\therefore p=q=\frac{1}{2}$ and $n=8$

$$\therefore P(2 \text{ success})={}^8C_2\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^2=\frac{28}{2^8}=\frac{28}{256}$$

10. **(b)** For a particular house being selected

Probability $=\frac{1}{3}$

P (all the persons apply for the same house)

$$=\left(\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\right)3=\frac{1}{9}.$$

11. **(c)** According to Poission distribution, prob. of getting k successes is

$$P(x=k)=e^{-\lambda}\frac{\lambda^k}{k!}\quad P(x \ge 2)=1-P(x=0)-P(x=1)$$

$$=1-e^{-\lambda}-e^{-\lambda}\left(\frac{\lambda}{1!}\right)=1-\frac{3}{e^2}.$$

12. **(c)** $P(\overline{A \cup B})=\frac{1}{6}, P(A \cap B)=\frac{1}{4}$ and $P(\overline{A})=\frac{1}{4}$

$$\Rightarrow P(A \cup B)=\frac{5}{6}\quad P(A)=\frac{3}{4}$$

Also $\Rightarrow P(A \cup B)=P(A)+P(B)-P(A \cap B)$

$$\Rightarrow P(B)=\frac{5}{6}-\frac{3}{4}+\frac{1}{4}=\frac{1}{3}$$

$$\Rightarrow P(A)\ P(B)=\frac{3}{4}-\frac{1}{3}=\frac{1}{4}=P(A \cap B)$$

Hence A and B are independent but not equally likely.

13. **(d)** $P(X=r)=\frac{e^{-m}m^r}{r!}$
P (at most 1 phone call)
$=P(X \le 1)=P(X=0)+P(X=1)$

$$=e^{-5}+5\times e^{-5}=\frac{6}{e^5}$$

14. **(d)** Given : Probability of aeroplane I, scoring a target correctly i.e., $P(I)=0.3$ probability of scoring a target correctly by aeroplane II, i.e. $P(II)=0.2$

$\therefore P(\bar{I})=1-0.3=0.7$ $\therefore$ The required probability

$=P(\bar{I} \cap II)=P(\bar{I}).P(II)=0.7\times0.2=0.14$

15. **(b)** A pair of fair dice is thrown, the sample space $S=(1,1)$, $(1,2)(1,3)....=36$
Possibility of getting 9 are $(5,4), (4,5), (6,3), (3,6)$
$\therefore$ Probability of getting score 9 in a single throw

$$=\frac{4}{36}=\frac{1}{9}$$

$\therefore$ Probability of getting score 9 exactly twice

$$={}^3C_2\times\left(\frac{1}{9}\right)^2.\left(1-\frac{1}{9}\right)=\frac{3!}{2!}\times\frac{1}{9}\times\frac{1}{9}\times\frac{8}{9}$$

$$=\frac{3.2!}{2!}\times\frac{1}{9}\times\frac{1}{9}\times\frac{8}{9}=\frac{8}{243}$$

16. **(b)** $P(A)=1/4, P(A/B)=\frac{1}{2}, P(B/A)=2/3$

By conditional probability,

$P(A\cap B)=P(A)P(B/A)=P(B)P(A/B)$

$\Rightarrow \frac{1}{4}\times\frac{2}{3}=P(B)\times\frac{1}{2}\Rightarrow P(B)=\frac{1}{3}$

17. **(c)** $A\equiv$ number is greater than 3 $\Rightarrow P(A)=\frac{3}{6}=\frac{1}{2}$

$B\equiv$ number is less than 5 $\Rightarrow P(B)=\frac{4}{6}=\frac{2}{3}$

$A\cap B\equiv$ number is greater than 3 but less than 5.

$\Rightarrow P(A\cap B)=\frac{1}{6}$

$\therefore P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{1}{2}+\frac{2}{3}-\frac{1}{6}=\frac{3+4-1}{6}=1$

18. **(d)** We have

$P(x\geq 1)\geq\frac{9}{10}\Rightarrow 1-P(x=0)\geq\frac{9}{10}$

$\Rightarrow 1-{}^nC_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^n\geq\frac{9}{10}$

$\Rightarrow 1-\frac{9}{10}\geq\left(\frac{3}{4}\right)^n \Rightarrow \left(\frac{3}{4}\right)^n\leq\left(\frac{1}{10}\right)$

Taking log to the base 3/4, on both sides, we get

$n\log_{3/4}\left(\frac{3}{4}\right)\geq\log_{3/4}\left(\frac{1}{10}\right)$

$\Rightarrow n\geq-\log_{3/4}10=\frac{-\log_{10}10}{\log_{10}\left(\frac{3}{4}\right)}=\frac{-1}{\log_{10}3-\log_{10}4}$

$\Rightarrow n\geq\frac{1}{\log_{10}4-\log_{10}3}$

19. **(d)** Let $A\equiv$ Sum of the digits is 8

$B\equiv$ Product of the digits is 0

Then A = {08, 17, 26, 35, 44}

B = {00, 01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 20, 30, 40,}

$A\cap B=\{08\}$

$\therefore$ P (A/B) = $\frac{P(A\cap B)}{P(B)}=\frac{1/50}{14/50}=\frac{1}{14}$

20. **(b)** $n(S)={}^{20}C_4$

Statement-1:

common difference is 1; total number of cases = 17
common difference is 2; total number of cases = 14
common difference is 3; total number of cases = 11
common difference is 4; total number of cases = 8
common difference is 5; total number of cases = 5
common difference is 6; total number of cases = 2

Prob. = $\frac{17+14+11+8+5+2}{{}^{20}C_4}=\frac{1}{85}$

Statement -2 is false, because common difference can be 6 also.

21. **(a)** $n(S)={}^9C_3,\ n(E)={}^3C_1\times{}^4C_1\times{}^2C_1$

Probability = $\frac{3\times4\times2}{{}^9C_3}=\frac{24\times3!}{9!}\times6!=\frac{24\times6}{9\times8\times7}=\frac{2}{7}$

22. **(b)** p (at least one failure) $\geq\frac{31}{32}$

$\Rightarrow 1-p$ (no failure) $\geq\frac{31}{32}$

$\Rightarrow 1-p^5\geq\frac{31}{32}\Rightarrow p^5\leq\frac{1}{32}\Rightarrow p\leq\frac{1}{2}$

But $p\geq 0$,

Hence p lies in the interval $\left[0,\frac{1}{2}\right]$.

23. **(a)** In this case, $P\left(\frac{C}{D}\right)=\frac{P(C\cap D)}{P(D)}=\frac{P(C)}{P(D)}$

Where, $0\leq P(D)\leq 1$, hence $P\left(\frac{C}{D}\right)\geq P(C)$

24. **(b)** Given sample space = {1,2,3,.....,8}

Let Event

A : Maximum of three numbers is 6.

B : Minimum of three numbers is 3.

This is the case of conditional probability

We have to find P (minimum) is 3 when it is given that P (maximum) is 6.

$\therefore P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}=\frac{{}^2C_1/{}^8C_3}{{}^5C_2/{}^8C_3}=\frac{{}^2C_1}{{}^5C_2}=\frac{2}{10}=\frac{1}{5}$

25. **(c)** $p=P$ (correct answer), $q=P$ (wrong answer)

$\Rightarrow p=\frac{1}{3}, q=\frac{2}{3}, n=5$

By using Binomial distribution

Required probability = ${}^5C_4\left(\frac{1}{3}\right)^4\cdot\frac{2}{3}+{}^5C_5\left(\frac{1}{3}\right)^5$

$=5\cdot\frac{2}{3^5}+\frac{1}{3^5}=\frac{11}{3^5}$

26. **(a)** Given

$P(\overline{A\cup B})=\frac{1}{6}\Rightarrow P(A\cup B)=1-\frac{1}{6}=\frac{5}{6}$

$P(\overline{A})=\frac{1}{4}\Rightarrow P(A)=1-\frac{1}{4}=\frac{3}{4}$

We know

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow \frac{5}{6}=\frac{3}{4}+P(B)-\frac{1}{4}\quad\left(\because P(A\cap B)=\frac{1}{4}\right)\Rightarrow P(B)=\frac{1}{3}$

$\because\ P(A)\neq P(B)$ so they are not equally likely.

Also $P(A)\times P(B)=\frac{3}{4}\times\frac{1}{3}=\frac{1}{4}=P(A\cap B)$

So A & B are independent.

27. **(c)** **Note:-** The question should state '3 different' boxes instead of '3 identical boxes' and one particular box has 3 balls. Then the solution would be:

Required probability = $\frac{{}^{12}C_3\times2^9}{3^{12}}=\frac{55}{3}\left(\frac{2}{3}\right)^{11}$

28. **(b)** $P(E_1)=\frac{1}{6}$; $P(E_2)=\frac{1}{6}$; $P(E_3)=\frac{1}{2}$

$P(E_1\cap E_2)=\frac{1}{36}, P(E_2\cap E_3)=\frac{1}{12}, P(E_1\cap E_3)=\frac{1}{12}$

And $P(E_1\cap E_2\cap E_3)=0\neq P(E_1).P(E_2).P(E_3)$

$\Rightarrow E_1, E_2, E_3$ are not independent.